Complex Numbers Summary

- When dealing with square roots of negative numbers, put in terms of j first.
- Add/Subtract/Multiply/Divide in Rectangular form.
  - Add example: $(-4-j)+(-6+2j)=-10+j$

  - Subtract example: $(-6+2j)-(3-5j)$

    $$(-6+2j)+(-3+5j)=-9+7j$$

  - Multiply (think algebraically-FOIL)

    Example: $(-2+3j)(7+6j)=-14-12j+21j+18j^2$

    $$=-14+9j+(18)(-1)$$

    $$=-32+9j$$

  - Divide (Use conjugate, multiply by numerator and denominator)

    Example: $\dfrac{9-8j}{-2-4j}\rightarrow\dfrac{(9-8j)}{(-2-4j)}\bullet\dfrac{(-2+4j)}{(-2+4j)}$

    $$=\frac{-18+36j+16j-32j^2}{4-16j^2}$$

    $$=\frac{14+52j}{20}=\frac{7+26j}{10}$$

- Convert rectangular to polar form
  - Example: $3-4j$

    Step 1: Sketch it:

    Step 2: Find r $\quad r=\sqrt{(3)^2+(-4)^2}=5$

    Step 3: Find θ_{ref} $\quad \tan\theta_{ref}=4/3$

    Step 4: Depending on quadrant, find θ $\quad \theta_{ref}=53.1°$

    $$\theta=360°-53.1°=306.9°$$

    Step 5: Write in polar form $\quad 5(\cos 306.9°+j\sin 306.9°)$

    OR

    $$5\angle 306.9°$$

- Convert from polar to exponential
  - Example: $4\angle 136° = 4(\cos 136° + j\sin 136°)$

    Convert angle θ to radians $\theta = 136° \times \pi / 180° = 2.37$

    Therefore, exponential form $\rightarrow 4e^{2.37j}$
- Convert polar back to rectangular
  - Example: $4(\cos 136° + j\sin 136°)$ use distributive property on this

    $4(\cos 136°) + 4(j\sin 136°)$

    $-2.88 + 2.78j$
- Multiply/Divide/Raise to power/Roots in Polar & Exponential Form
  - When multiplying, multiply the r values and add angle θ's.

    Example: $[3.7(\cos 57° + j\sin 57°)][2.4(\cos 122° + j\sin 122°)]$

    $= (3.7)(2.4)[\cos(57° + 122°) + \sin(57° + 122°)]$

    $= 8.88(\cos 179° + j\sin 179°)$ OR $8.88\angle 179°$
  - When dividing, divide the r values and subtract angle θ's

    Example: $\dfrac{4.2(\cos 225° + j\sin 225°)}{2.1(\cos 46° + j\sin 46°)}$

    $= \dfrac{4.2}{2.1}[\cos(225° - 46°) + j\sin(225° - 46°)]$

    $= 2(\cos 179° + j\sin 179°)$ OR $2\angle 179°$
  - Raise to a power, raise the r value to the power, and multiply angle θ by the power.

    Example: $[2.3(\cos 120° + j\sin 120°)]^8$

    $= 2.3^8[\cos(8)(120°) + j\sin(8)(120°)]$

    $= 783.11(\cos 960° + j\sin 960°) \rightarrow$ take 960° and subtract 360° until you have a value between 0° and 360°.

    $= 783.11(\cos 240° + j\sin 240°)$

- Taking roots (like raising to a power)

  Example: Find the root of $-1 \rightarrow 1(\cos 180° + j \sin 180°)$

  $= 1(\cos 180° + j \sin 180°)^{\frac{1}{3}}$

  $= 1^{\frac{1}{3}}(\cos \frac{180°}{3} + j \sin \frac{180°}{3})$

  $= 1(\cos 60° + j \sin 60°)$ →this is the first root.

  There are 3 because

  you are taking the third root.

  Next, $\frac{360°}{3} =$ **120° intervals**

  2nd root $\rightarrow 1[\cos(60° + 120°) + j \sin(60° + 120°)]$

  $= 1(\cos 180° + j \sin 180°)$

  3rd root $\rightarrow 1(\cos 300° + j \sin 300°)$ (we added 120° from the angle of the previous root)

Review Test for Complex Numbers

Perform the indicated operations and express the final answer in the simplest rectangular form.

1) $(12+7j)+(-8+6j)$

2) $(-4-2j)-(-6-\sqrt{-49})$

3) $(-5+3j)(8-4j)$

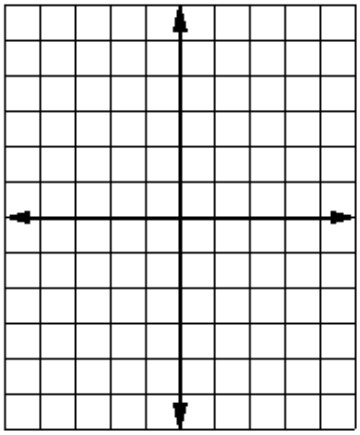
4) $\dfrac{3+\sqrt{-4}}{4-j}$

Find the values of x and y which make the equations valid.

5) $3x-2j=yj-2$

Perform the indicated operation graphically, check your answer algebraically.

6) $(1+4j)-(-3-3j)$



Give the polar and exponential forms of each of the complex numbers:

7) $6-2j$

8) $-4j$

Give the rectangular form of each number:

9) $20(\cos 160^\circ + j\sin 160^\circ)$

10) $44.47e^{6.046j}$

In the following exercises, perform the indicated operation. Leave the results in polar form.

11) $(0.1254\angle 172.38°)(27.17\angle 204.34°)$

12) $\dfrac{18(\cos 403° + j\sin 403°)}{4(\cos 192° + j\sin 192°)}$

13) $4.944\angle 327.49° + 8.009\angle 7.37°$

14) $[2(\cos 16° + j\sin 16°)]^{10}$

In the following exercise, change each number to polar form and then perform the indicated operation. Express the final result in both polar and rectangular forms. Check by performing the same operation in rectangular form.

15) $\dfrac{(5+5j)^4}{(-1-j)^6}$

16) Find the fourth root of $-j$