Section 5.4 (Solving Systems of Two Linear Equations in Two Unknowns by Determinants)

Systems of equations can also be solved in a very systematic way using a procedure involving determinants. This method was discovered by a mathematician named "Cramer" and it is called Cramer's rule.

The objective for this section is to:

- Solve a system of linear equations using determinants

The mathematician named Cramer discovered that when working with a system of equation in two variables that is in standard form of:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

If we multiply the first equation by b_2 and the second equation by b_1, we get :

$$a_1b_2x + b_1b_2y = c_1b_2$$
$$a_2b_1x + b_2b_1y = c_2b_1$$

and subtracting the second one from the first, finish the rest by hand.

In order to solve a system of equations using the pattern that Cramer discovered we must know how to evaluate a determinant.

Evaluate the given determinants

1.) $\begin{vmatrix} 2 & 8 \\ -3 & 5 \end{vmatrix}$

2.) $\begin{vmatrix} -6 & -2 \\ -8 & 1 \end{vmatrix}$

Section 5.4 (Solving Systems of Two Linear Equations in Two Unknowns by Determinants)

Use Determinants to solve the given systems of equations.

3.) $\begin{aligned} x+2y&=7 \\ x-y&=-2 \end{aligned}$

4.) $\begin{aligned} 2x-y&=-17 \\ x+5y&=41 \end{aligned}$

5.) $\begin{aligned} 8x+12y&=16 \\ 6x+9y&=-5 \end{aligned}$

6.) Four pine seedlings and 3 fir seedlings cost $29.00. Two pine seedlings and 7 fir seedlings cost $42.00. Determine the cost of each type of seedling.

7.) An executive nearing retirement made two investments totaling $15,000. In one year, these investments yielded $1432 in simple interest. Part of the money was invested at 9% and the rest at 10%. How much was invested at each rate?

Section 5.4 (Solving Systems of Two Linear Equations in Two Unknowns by Determinants)

We've been doing many problems throughout this section that involved systems of equations. We will illustrate a few more examples.

The objective for this section is to:

- Set up systems of equations from application problems
- Use appropriate methods to solve systems of linear equations

1. The sum of two voltages is 100 V. If the higher voltage is doubled and the other halved, the sum becomes 155 V. What are the voltages?

2. A company maintains two separate accounts for the investment of $45,000. Part of this amount is invested at 6% and the remainder at 8%. If the total annual income is $3400, how much is invested at teach rate?

3. It takes a machine 2 hours to process six items of one type and two items of a second type. It takes the same machine 4 hours to process five items of each type. Find the time required to process each of the two types of items.

4. Suppose that 120 children and 140 adults attend a show for which the total receipts are $530. If a parent and two young children pay $5.50, how much are the adult tickets and how much are the children's tickets?

5. How many liters of a mixture containing 70% alcohol should be added to a mixture containing 20% alcohol to give 16L of a mixture containing 50% alcohol?

6. A company can hire a total of 18 telecommunication specialists and technicians for a special project. The specialists earn $55/hour whereas the technicians earn $37.50/hour. How many specialists and technicians can the company hire if they have budgeted $797.50 per hour for salaries for this project?