Section 5.3 (Solving Systems of Two Linear Equations in Two Unknowns Algebraically)

Systems of equations can be solved in a variety of ways and without a graphing calculator, I typically do not turn to graphical means of solution. This is because in real world problems the answers don't tend to turn out to be integers all the time and if you graph by hand finding the exact solution is challenging. In this section we will learn two more methods of solving a system of equations.

The objectives for this section include:

- Solve a system of linear equations using the substitution method
- Solve a system of linear equations using the addition method (often called elimination)

Procedure for solving a system using the Substitution Method

Step 1: Isolate one of the variables in ONE of the equations
Step 2: Substitute the expression from step 1 into the OTHER equation.
Step 3: Solve that second equation for the remaining variable
Step 4: Substitute the value found for the variable in Step 3 into the first equation to determine the value of the other variable.
Step 5: Check the solutions for the variables in BOTH of the original equations.

Solve these systems using the substitution method:

1.) $\begin{array}{l} y = x + 2 \\ x + 2y = 7 \end{array}$

2.) $\begin{array}{l} 5x + 6y = 14 \\ -3y + x = 7 \end{array}$

3.) $$\begin{aligned} t &= 4 - 2s \\ t + 2s &= 6 \end{aligned}$$

4.) $$\begin{aligned} 12x - 6y &= -15 \\ -4x + 2y &= 5 \end{aligned}$$

5.) Under certain conditions, the tensions T_1 and T_2 (in newtons) supporting a derrick are found by solving the equations

$$0.68T_1 + 0.57T_2 = 750$$
$$0.73T_1 - 0.82T_2 = 0$$

6.) Staples recently sold a wirebound graph-paper notebook for $2.50 and a college-ruled notebook made of recycled paper for $2.30. At the start of a recent spring semester, a combination of 50 of these notebooks was sold for a total of $118.60. How many of each type were sold?

7.) The sum of two numbers is 50. The first number is 25% of the second number. What are the numbers?

8.) Determine *a* and *b* for which $(-4,-3)$ is a solution of the system:

$$ax+by=-26$$
$$bx-ay=7$$

9.) Describe a procedure that can be used to write an inconsistent system of equations.

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Procedure for solving a system using the Addition or Elimination Method

Step 1: Rewrite each equation in standard form $ax+by=c$

Step 2: If necessary, multiply each equation by a constant so that the coefficients of one of the variable will add up to zero.

Step 3: Add the two equations to eliminate a variable

Step 4: Solve the resulting equation for the other variable.

Step 5: Substitute this value into one of the original equations and solve for the other variable

Step 5: Check the solutions for the variables in BOTH of the original equations.

Solve these systems using the addition method:

10.) $\begin{matrix} x+y=11 \\ 2x-y=1 \end{matrix}$

11.) $\begin{matrix} 4y=x+7 \\ x+3y=7 \end{matrix}$

12.) $2x+2y=2$
$3x-\ y=1$

13.) $5x+3y=19$
$2x-5y=11$

14.) $10a+6b=8$
$5a+3b=2$

15.)
$$\frac{2x}{3}+\frac{3y}{4}=\frac{11}{12}$$
$$\frac{x}{3}+\frac{7y}{18}=\frac{1}{2}$$

16.)
$$\frac{1}{2}x-\frac{1}{6}y=3$$
$$\frac{2}{5}x+\frac{1}{2}y=2$$

17.) One line printer can produce *x* lines per minute; a second line printer can produce *y* lines per minute. They print 7500 lines if the first prints for 2 minutes and the second prints for 1 minute.

They can print 9000 lines if the second prints for 2 minutes and the first for 1 minute. Determine the printing rates for each printer.

18.) An office building with 54 offices has two types of offices. One type of office rents for \$900/month; the other type rents for \$1250/month. If all of the offices are occupied and the total rental income is \$55,600/month, determine how many of each office type there is.

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In each of the following exercises match the system listed with the choice from the column on the right that would be a subsequent step in solving the system.

1. ______ $\begin{array}{l} -2x+3y=7 \\ 2x+5y=-8 \end{array}$

2. ______ $\begin{array}{l} x=4y-5 \\ 5x+7y=2 \end{array}$

3. ______ $\begin{array}{l} 3x+4y=13 \\ 4x-2y=5 \end{array}$

4. ______ $\begin{array}{l} 8x+6y=-15 \\ 5x-3y=8 \end{array}$

5. ______ $\begin{array}{l} y=4x-7 \\ 6x+3y=19 \end{array}$

6. ______ $\begin{array}{l} y=\frac{2}{3}x+4 \\ y=-\frac{1}{5}x+4 \end{array}$

a) $\begin{array}{l} 3x+4y=13 \\ 8x-4y=10 \end{array}$

b) The lines intersect at $(0,4)$

c) $6x+3(4x-7)=19$

d) $8y=-1$

e) $5(4y-5)+7y=2$

f) $\begin{array}{l} 8x+6y=-15 \\ 10x-6y=16 \end{array}$