Section 2.1
Basics of Functions and Their Graphs

Say *WHAT???*

You may have noticed that mathematical notation occasionally can have more than one meaning depending on the context.

For example, $(-3, 6)$ could refer to the ordered pair where $x = -3$ and $y = 6$, or it could refer to the open interval $-3 < x < 6$.

Similarly, in this section of the textbook, we will use the notation, $f(x)$.
It may surprise you to find out that it does *not* mean to multiply "f times x."

It will be important for you to gain an understanding of what this notation *does* mean as you work through this essential concept of "functions."

Objective #1: Find the domain and range of a relation.

✔ *Solved Problem #1*	✎ *Pencil Problem #1* ✐
1. Find the domain and range of the relation: {(0, 187), (1, 212), (2, 225), (3, 238)}	**1.** Find the domain and range of the relation: {(3, 4), (3, 5), (4, 4), (4, 5)}

The domain is the set of all first components.
Domain:
{0, 1, 2, 3}.

The range is the set of all second components.

Range:
{187, 212, 225, 238}.

Objective #2: Determine whether a relation is a function.

✔ *Solved Problem #2*	✎ *Pencil Problem #2* ✐
2a. Determine whether the relation is a function: $\{(1,2),(3,4),(5,6),(5,8)\}$	**2a.** Determine whether the relation is a function: {(3, 4), (3, 5), (4, 4), (4, 5)}

5 corresponds to both 6 and 8. If any element in the domain corresponds to more than one element in the range, the relation is not a function.

Thus, the relation is not a function.

2b. Determine whether the relation is a function:
$$\{(1,2),(3,4),(6,5),(8,5)\}$$

Every element in the domain corresponds to exactly one element in the range. No two ordered pairs in the given relation have the same first component and different second components.

Thus, the relation is a function.

2b. Determine whether the relation is a function:
$$\{(-3,-3),(-2,-2),(-1,-1),(0,0)\}$$

Objective #3: Determine whether an equation represents a function.

✔ *Solved Problem #3*

3. Solve each equation for y and then determine whether the equation defines y as a function of x.

3a. $2x + y = 6$

Subtract $2x$ from both sides to solve for y.
$$2x + y = 6$$
$$2x - 2x + y = 6 - 2x$$
$$y = 6 - 2x$$

For each value of x, there is only one value of y, so the equation defines y as a function of x.

✎ *Pencil Problem #3*

3. Solve each equation for y and then determine whether the equation defines y as a function of x.

3a. $x^2 + y = 16$

3b. $x^2 + y^2 = 1$

Subtract x^2 from both sides and then use the square root property to solve for y.
$$x^2 + y^2 = 1$$
$$x^2 - x^2 + y^2 = 1 - x^2$$
$$y^2 = 1 - x^2$$
$$y = \pm\sqrt{1 - x^2}$$

For values of x between -1 and 1, there are two values of y. For example, if $x = 0$, then $y = \pm 1$. Thus, the equation does not define y as a function of x.

3b. $x = y^2$

Objective #4: Evaluate a function.	
✔ *Solved Problem #4*	✐ *Pencil Problem #4*✎

4. If $f(x) = x^2 - 2x + 7$, evaluate each of the following.

4a. $f(-5)$

Substitute -5 for x. Place parentheses around -5 when making the substitution.

$f(-5) = (-5)^2 - 2(-5) + 7$
$ = 25 + 10 + 7 = 42$

4b. $f(x + 4)$

Substitute $x + 4$ for x and then simplify. Place parentheses around $x + 4$ when making the substitution.

Use $(A + B)^2 = A^2 + 2AB + B^2$ to expand $(x+4)^2$ and the distributive property to multiply $-2(x + 4)$. Then combine like terms.

$f(x+4) = (x+4)^2 - 2(x+4) + 7$
$ = x^2 + 8x + 16 - 2x - 8 + 7$
$ = x^2 + 6x + 15$

4c. $f(-x)$

Substitute $-x$ for x. Place parentheses around $-x$ when making the substitution.

$f(-x) = (-x)^2 - 2(-x) + 7$
$ = x^2 + 2x + 7$

4. If $g(x) = x^2 + 2x + 3$, evaluate each of the following.

4a. $g(-1)$

4b. $g(x + 5)$

4c. $g(-x)$

Objective #5: Graph functions by plotting points.

| ✔ **Solved Problem #5** | ✎ **Pencil Problem #5** |

5. Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2. How is the graph of g related to the graph of f?
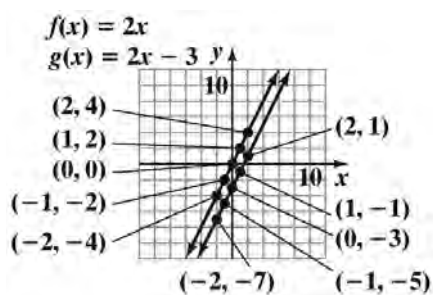
Make a table for $f(x) = 2x$:

x	$f(x) = 2x$	(x, y)
-2	$f(-2) = 2(-2) = -4$	$(-2, -4)$
-1	$f(-1) = 2(-1) = -2$	$(-1, -2)$
0	$f(0) = 2(0) = 0$	$(0, 0)$
1	$f(1) = 2(1) = 2$	$(1, 2)$
2	$f(2) = 2(2) = 4$	$(2, 4)$

Make a table for $g(x) = 2x - 3$:

| x	$g(x) = 2x - 3$	(x, y)
-2	$g(-2) = 2(-2) - 3 = -7$	$(-2, -7)$
-1	$g(-1) = 2(-1) - 3 = -5$	$(-1, -5)$
0	$g(0) = 2(0) - 3 = -3$	$(0, -3)$
1	$g(1) = 2(1) - 3 = -1$	$(1, -1)$
2	$g(2) = 2(2) - 3 = 1$	$(2, 1)$

Plot the points and draw the lines that pass through them.

The graph of g is the graph of f shifted down by 3 units.

5. Graph the functions $f(x) = |x|$ and $g(x) = |x| - 2$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2. How is the graph of g related to the graph of f?

Objective #6: Use the vertical line test to identify functions.

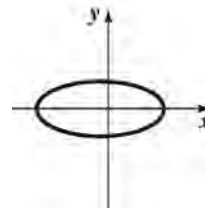| ✔ *Solved Problem #6*	✎ *Pencil Problem #6*
6. Use the vertical line test to determine if the graph represents *y* as a function of *x*.	**6.** Use the vertical line test to determine if the graph represents *y* as a function of *x*.
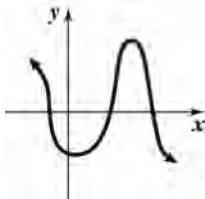
The graph passes the vertical line test and thus *y* is a function of *x*.

Objective #7: Obtain information about a function from its graph.

✔ *Solved Problem #7*	✎ *Pencil Problem #7*
7a. The following is the graph of *g*.	**7a.** The following is the graph of *g*.
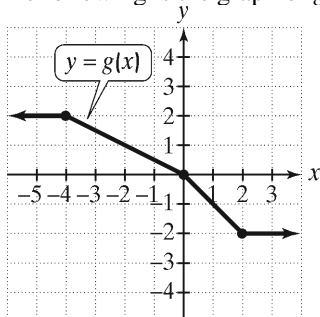
Use the graph to find $g(-20)$.

The graph indicates that to the left of $x = -4$, the graph is at a constant height of 2.

Thus, $g(-20) = 2$.
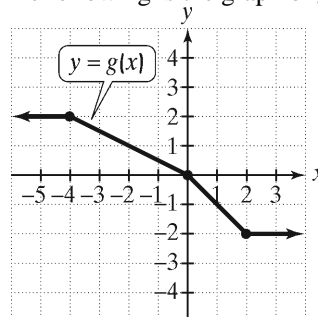
Use the graph to find $g(-4)$.

7b. Use the graph from *Problem 7a* above to find the value of *x* for which $g(x) = -1$.
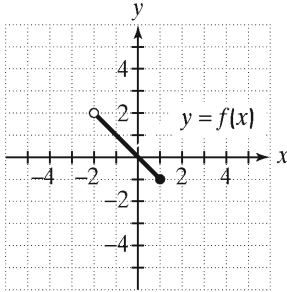
$g(1) = -1$

The height of the graph is -1 when $x = 1$.

7b. Use the graph from *Problem 7a* above to find the value of *x* for which $g(x) = 1$.

Objective #8: Identify the domain and range of a function from its graph.

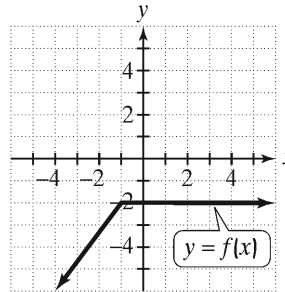| ✔ *Solved Problem #8*	✎ *Pencil Problem #8*
8. Use the graph of the function to identify its domain and its range.	**8.** Use the graph of the function to identify its domain and its range.

Inputs on the *x*-axis extend from –2, excluding –2, to 1, including 1.
The domain is $(-2,1]$.

Outputs on the *y*-axis extend from –1, including –1, to 2, excluding 2.
The range is $[-1,2)$.

Objective #9: Identify intercepts from a function's graph.

| ✔ *Solved Problem #9*	✎ *Pencil Problem #9*
9. True or false: The graph of a function may cross the *y*-axis several times, so the graph may have more than one *y*-intercept.	**9.** True or false: The graph of a function may cross the *x*-axis several times, so the graph may have more than one *x*-intercept.

False. Since each point on the *y*-axis has *x*-coordinate 0 and a function may have only one *y*-value for each *x*-value, the graph of a function has at most one *y*-coordinate.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. Domain: {3, 4}. Range: {4, 5}. *(2.1 #3)*   **2a.** not a function *(2.1 #3)*   **2b.** function *(2.1 #7)*

3a. $y = 16 - x^2$; *y* is a function of *x*. *(2.1 #13)*   **3b.** $y = \pm\sqrt{x}$; *y* is not a function of *x*. *(2.1 #17)*

4a. 2 *(2.1 #29a)*   **4b.** $x^2 + 12x + 38$ *(2.1 #29b)*   **4c.** $x^2 - 2x + 3$ *(2.1 #29c)*
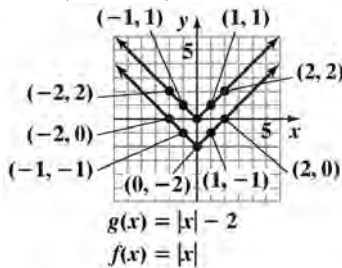
5.   The graph of *g* is the graph of *f* shifted down 2 units. *(2.1 #45)*

6. not a function *(2.1 #59)*   **7a.** 2 *(2.1 #71)*   **7b.** −2 *(2.1 #75)*

8. Domain: $(-\infty, \infty)$. Range: $(-\infty, -2]$ *(2.1 #87)*   **9.** True *(2.1 #77)*

Section 2.2
More on Functions and Their Graphs


Can I Really Tell That from a Graph?

Graphs provide a visual representation of how a function changes over time. Many characteristics of a function are much more evident from the function's graph than from the equation that defines the function.

We can use graphs to determine for what years the fuel efficiency of cars was increasing and decreasing and at what age men and women attain their maximum percent body fat. A graph can even help you understand your cellphone bill better.

Objective #1: Identify intervals on which a function increases, decreases, or is constant.

✔ *Solved Problem #1*

1. State the intervals on which the given function is increasing, decreasing, or constant.

$f(x) = x^3 - 3x$
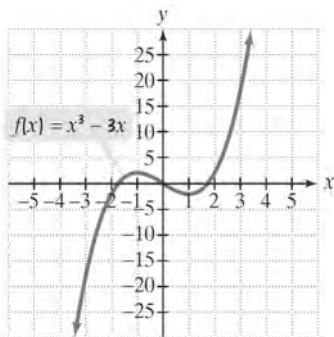
The intervals are stated in terms of *x*-values.
When we start at the left and follow along the graph, at first the graph is going up. This continues until $x = -1$.
The function is increasing on the interval $(-\infty, -1)$.

At $x = -1$, the graph turns and moves downward until we get to $x = 1$. The function is decreasing on the interval $(-1, 1)$.

At $x = 1$, the graph turns again and continues in an upward direction. The function is increasing on the interval $(1, \infty)$.

✎ *Pencil Problem #1*

1. State the intervals on which the given function is increasing, decreasing, or constant.
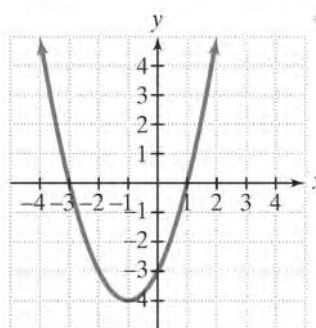
Objective #2: Use graphs to locate relative maxima or minima.

✔ *Solved Problem #2*

2. Look at the graph in Solved Problem #1. Locate values at which the function *f* has any relative maxima or minima. What are these relative maxima or minima?

The graph has a turning point at $x = -1$. The value of $f(x)$ or y at $x = -1$ is greater than the values of $f(x)$ for values of x near -1 (for values of x between -2 and 0, for example). Thus, *f* has a relative maximum at $x = -1$. The relative maximum is the value of $f(x)$ or y corresponding to $x = -1$. Using the equation in the graph, we find that $f(-1) = (-1)^3 - 3(-1) = 2$. We say that *f* has a relative maximum of 2 at $x = -1$.

The graph has a second turning point at $x = 1$. The value of $f(x)$ or y at $x = 1$ is less than the values of $f(x)$ for values of x near 1 (for values of x between 0 and 2, for example). Thus, *f* has a relative minimum at $x = 1$. The relative minimum is the value of $f(x)$ or y corresponding to $x = 1$. Using the equation in the graph, we find that $f(1) = (1)^3 - 3(1) = -2$. We say that *f* has a relative minimum of -2 at $x = 1$.

Note that the relative maximum occurs where the functions changes from increasing to decreasing and the relative minimum occurs where the graph changes from decreasing to increasing.

✎ *Pencil Problem #2*

2. The graph of a function *f* is given below. Locate values at which the function *f* has any relative maxima or minima. What are these relative maxima or minima? Read *y*-values from the graph, as needed, since the equation is not given.
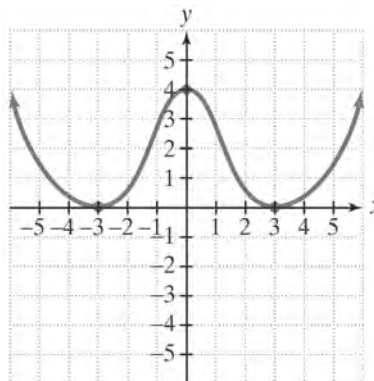
Objective #3: Test for symmetry.

✔ *Solved Problem #3*

3. Determine whether the graph of each equation is symmetric with respect to the *y*-axis, the *x*-axis, or the origin.

3a. $y = x^2 - 1$

To test for *y*-axis symmetry, replace *x* with $-x$.

$y = (-x)^2 - 1$

$y = x^2 - 1$

The result is the original equation, so the graph is symmetric with respect to the *y*-axis.

✎ *Pencil Problem #3*

3. Determine whether the graph of each equation is symmetric with respect to the *y*-axis, the *x*-axis, or the origin.

3a. $y = x^2 + 6$

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To test for x-axis symmetry, replace y with $-y$.

$$-y = x^2 - 1$$
$$y = -x^2 + 1$$

The result is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

To test for origin symmetry, replace x with $-x$ and y with $-y$.

$$-y = (-x)^2 - 1$$
$$-y = x^2 - 1$$
$$y = -x^2 + 1$$

The result is not equivalent to the original equation, so the graph is not symmetric with respect to the origin.

The graph of $y = x^2 - 1$ is symmetric with respect to the y-axis only.

3b. $y^5 = x^3$

To test for y-axis symmetry, replace x with $-x$.

$$y^5 = (-x)^3$$
$$y^5 = -x^3$$

The result is not equivalent to the original equation, so the graph is not symmetric with respect to the y-axis.

To test for x-axis symmetry, replace y with $-y$.

$$(-y)^5 = x^3$$
$$-y^5 = x^3$$
$$y^5 = -x^3$$

The result is not equivalent to the original equation, so the graph is not symmetric with respect to the x-axis.

To test for origin symmetry, replace x with $-x$ and y with $-y$.

$$(-y)^5 = (-x)^3$$
$$-y^5 = -x^3$$
$$y^5 = x^3$$

The result is the original equation, so the graph is symmetric with respect to the origin.

The graph of $y^5 = x^3$ is symmetric with respect to the origin only.

3b. $x^2 + y^2 = 100$

Objective #4: Identify even or odd functions and recognize their symmetries.

✔ *Solved Problem #4*	✏ *Pencil Problem #4*
4. Determine whether each of the following functions is even, odd, or neither.	**4.** Determine whether each of the following functions is even, odd, or neither.

4a. $f(x) = x^2 + 6$

Replace x with $-x$.

$f(-x) = (-x)^2 + 6 = x^2 + 6 = f(x)$

The function did not change when we replaced x with $-x$. The function is even.

4a. $f(x) = x^3 + x$

4b. $g(x) = 7x^3 - x$

Replace x with $-x$.

$g(-x) = 7(-x)^3 - (-x) = -7x^3 + x = -g(x)$

Each term of the equation defining the function changed sign when we replaced x with $-x$. The function is odd.

4b. $g(x) = x^2 + x$

4c. $h(x) = x^5 + 1$

Replace x with $-x$.

$h(x) = (-x)^5 + 1 = -x^5 + 1$

The resulting function is not equal to the original function, so the function is not even. Only the sign of one term changed, so the function is not odd. The function is neither even nor odd.

4c. $h(x) = x^2 - x^4$

Objective #5: Understand and use piecewise functions.
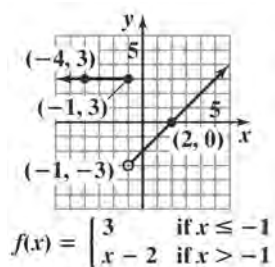
✔ *Solved Problem #4*	✏ *Pencil Problem #4*

5. Graph the piecewise function defined by

$$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x-2 & \text{if } x > -1 \end{cases}$$

For $x \le -1$, the function value is always 3, so $(-4, 3)$ and $(-1, 3)$ are examples of points on the first piece of the graph.

For $x > -1$, we use $f(x) = x - 2$. We have points such as $(0, -2)$ and $(2, 0)$ on the graph. Note that this piece of the graph will approach the point $(-1, -3)$ but this point is not part of the graph. We use an open dot at $(-1, -3)$.

$$f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x-2 & \text{if } x > -1 \end{cases}$$

5. Graph the piecewise function defined by

$$f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$$

Objective #6: Find and simplify a function's difference quotient.

✔ *Solved Problem #6*	✏ *Pencil Problem #6*

6. Find and simplify the difference quotient for $f(x) = -2x^2 + x + 5$.

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{[-2(x+h)^2 + (x+h) + 5] - (-2x^2 + x + 5)}{h}$$

$$= \frac{-2(x^2 + 2xh + h^2) + x + h + 5 + 2x^2 - x - 5}{h}$$

$$= \frac{-2x^2 - 4xh - 2h^2 + x + h + 5 + 2x^2 - x - 5}{h}$$

$$= \frac{-4xh - 2h^2 + h}{h}$$
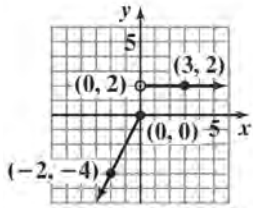
$$= \frac{h(-4x - 2h + 1)}{h}$$

$$= -4x - 2h + 1, \ h \ne 0$$

6. Find and simplify the difference quotient for $f(x) = x^2 - 4x + 3$.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. decreasing on $(-\infty, -1)$; increasing on $(-1, \infty)$ *(2.2 #1)*

2. relative minimum of 0 at $x = -3$; relative maximum of 4 at $x = 0$; relative minimum of 0 at $x = 3$ *(2.2 #13)*

3a. y-axis *(2.2 #17)*   **3b.** x-axis, y-axis, and the origin *(2.2 #27)*

4a. odd *(2.2 #37)*   **4b.** neither *(2.2 #39)*   **4c.** even *(2.2 #41)*

5.

$$f(x) = \begin{cases} 2x & \text{if } x \le 0 \\ 2 & \text{if } x > 0 \end{cases}$$ *(2.2 #61)*

6. $2x + h - 4, h \neq 0$ *(2.2 #77)*

Section 2.3
Linear Functions and Slope

Objective #1: Calculate a line's slope.

✔ *Solved Problem #1*

1. Find the slope of the line passing through $(4,-2)$ and $(-1,5)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{5-(-2)}{-1-4}$$
$$= \frac{7}{-5}$$
$$= -\frac{7}{5}$$

Pencil Problem #1

1. Find the slope of the line passing through $(-2,1)$ and $(2,2)$.

Objective #2: Write the point-slope form of the equation of a line.

✔ *Solved Problem #2*

2a. Write the point-slope form of the equation of the line with slope 6 that passes through the point $(2,-5)$. Then solve the equation for *y*.

Begin by finding the point-slope equation of a line.
$$y - y_1 = m(x - x_1)$$
$$y - (-5) = 6(x - 2)$$
$$y + 5 = 6(x - 2)$$

Now solve this equation for *y*.
$$y + 5 = 6(x - 2)$$
$$y + 5 = 6x - 12$$
$$y = 6x - 17$$

Pencil Problem #2

2a. Write the point-slope form of the equation of the line with slope -3 that passes through the point $(-2,-3)$. Then solve the equation for *y*.

2b. A line passes through the points $(-2,-1)$ and $(-1,-6)$. Find the equation of the line in point-slope form and then solve the equation for y.

Begin by finding the slope: $m = \dfrac{-6-(-1)}{-1-(-2)} = \dfrac{-5}{1} = -5$

Using the slope and either point, find the point-slope equation of a line.

$$y - y_1 = m(x - x_1) \qquad \text{or} \qquad y - y_1 = m(x - x_1)$$

$$y-(-1) = -5(x-(-2)) \qquad\qquad y-(-6) = -5(x-(-1))$$

$$y+1 = -5(x+2) \qquad\qquad\qquad y+6 = -5(x+1)$$

To obtain slope-intercept form, solve the above equation for y:

$$y+1 = -5(x+2) \quad \text{or} \quad y+6 = -5(x+1)$$

$$y+1 = -5x-10 \qquad\qquad y+6 = -5x-5$$

$$y = -5x-11 \qquad\qquad\quad y = -5x-11$$

2b. A line passes through the points $(-3,-1)$ and $(2,4)$. Find the equation of the line in point-slope form and then solve the equation for y.

Objective #3: Write and graph the slope-intercept form of the equation of a line.

✔ *Solved Problem #3*

3. Graph: $f(x) = \dfrac{3}{5}x+1$

The y-intercept is 1, so plot the point $(0,1)$.

The slope is $m = \dfrac{3}{5}$.

Find another point by going up 3 units and to the right 5 units.

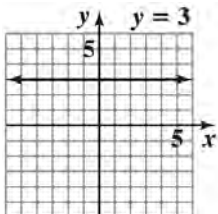Use a straightedge to draw a line through the two points.
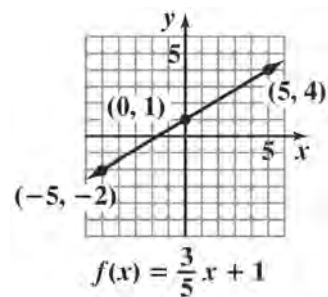
✎ *Pencil Problem #3*

3. Graph: $f(x) = \dfrac{3}{4}x-2$

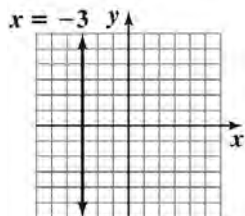| ***Objective #4:*** Graph horizontal or vertical lines.	
✔ *Solved Problem #4*	✎ *Pencil Problem #4* ✐

4a. Graph $y = 3$ in the rectangular coordinate system.

$y = 3$ is a horizontal line.

4a. Graph $y = -2$ in the rectangular coordinate system.

4b. Graph $x = -3$ in the rectangular coordinate system.

$x = -3$ is a vertical line.

4b. Graph $x = 5$ in the rectangular coordinate system.

Objective #5: Recognize and use the general form of a line's equation.

✔ *Solved Problem #5*	✏ *Pencil Problem #5*
5. Find the slope and *y*-intercept of the line whose equation is $3x + 6y - 12 = 0$.	**5.** Find the slope and *y*-intercept of the line whose equation is $2x + 3y - 18 = 0$.

Solve for *y*.

$3x + 6y - 12 = 0$

$$6y = -3x + 12$$

$$\frac{6y}{6} = \frac{-3x + 12}{6}$$

$$y = \frac{-3}{6}x + \frac{12}{6}$$

$$y = -\frac{1}{2}x + 2$$

The coefficient of *x*, $-\dfrac{1}{2}$, is the slope, and the constant term, 2, is the *y*-intercept.

Objective #6: Use intercepts to graph a linear function in standard form.

✔ *Solved Problem #6*	✏ *Pencil Problem #6*
6. Graph: $3x - 2y - 6 = 0$	**6.** Graph: $6x - 2y - 12 = 0$

Find the *x*–intercept by setting $y = 0$.

$3x - 2y - 6 = 0$

$3x - 2(0) - 6 = 0$

$$3x = 6$$

$$x = 2$$

Find the *y*–intercept by setting $x = 0$.

$3x - 2y - 6 = 0$

$3(0) - 2y - 6 = 0$

$$-2y = 6$$

$$y = -3$$

Plot the points and draw the line that passes through them.
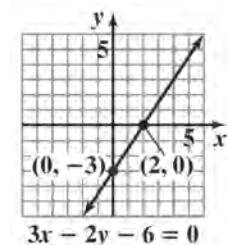
$3x - 2y - 6 = 0$

Objective #7: Model data with linear functions and make predictions.

✔ Solved Problem #7

7. The amount of carbon dioxide in the atmosphere, measured in parts per million, has been increasing as a result of the burning of oil and coal. The buildup of gases and particles is believed to trap heat and raise the planet's temperature. When the atmospheric concentration of carbon dioxide is 317 parts per million, the average global temperature is 57.04°F. When the atmospheric concentration of carbon dioxide is 354 parts per million, the average global temperature is 57.64°F.

Write a linear function that models average global temperature, $f(x)$, for an atmospheric concentration of carbon dioxide of x parts per million. Use the function to project the average global temperature when the atmospheric concentration of carbon dioxide is 600 parts per million.

Write the equation of the line through the points (317, 57.04) and (354, 57.64). First find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{57.64 - 57.04}{354 - 317} = \frac{0.6}{37} \approx 0.016$$

Use this slope and the point (317, 57.04) in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 57.04 = 0.016(x - 317)$$
$$y - 57.04 = 0.016x - 5.072$$
$$y = 0.016x + 51.968$$

Using function notation and rounding the constant, we have

$$f(x) = 0.016x + 52.0$$

To predict the temperature when the atmospheric concentration of carbon dioxide is 600 parts per million, find $f(600)$.

$$f(600) = 0.016(600) + 52.0 = 61.6$$

The model predicts an average global temperature of 61.6°F when the atmospheric concentration of carbon dioxide is 600 parts per million.

✎ Pencil Problem #7✎

7. The life expectancy for men born in 1980 is 70.0 years, and the life expectancy for men born in 2000 is 74.3 years. Let x represent the number of birth years after 1960 and y male life expectancy. Write a linear function that models life expectancy, $E(x)$, for American men born x years after 1960. Use the function to project the life expectancy of American men born in 2020.
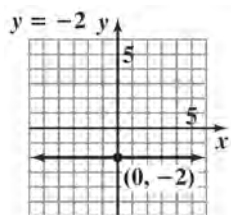
.

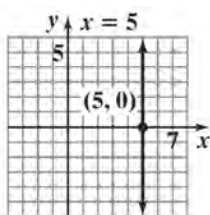### **Answers** for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $\dfrac{1}{4}$ *(2.3 #3)*

2a. $y+3=-3(x+2); y=-3x-9$ *(2.3 #15)*   **2b.** $y+1=1(x+3)$ or $y-4=1(x-2); y=x+2$ *(2.3 #29)*
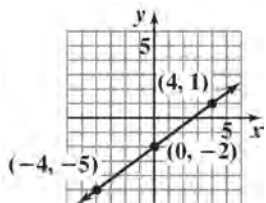
3.

$f(x)=\dfrac{3}{4}x-2$ *(2.3 #43)*

4a.

(2.3 #49)   **4b.**

(2.3 #52)

5. slope: $-\dfrac{2}{3}$; y-intercept: 6 *(2.3 #61)*
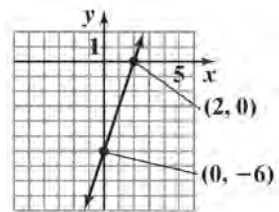
6.

$6x-2y-12=0$ *(2.3 #67)*

7. $E(x)=0.215x+65.7$; 78.6 years *(2.3 #89)*

Section 2.4
More on Slope

Objective #1: Find slopes and equations of parallel and perpendicular lines.

✔ *Solved Problem #1*	✏ *Pencil Problem #1* ✏
1a. Write an equation of the line passing through $(-2,5)$ and parallel to the line whose equation is $y = 3x+1$. Express the equation in point-slope form and slope-intercept form.	**1a.** Write an equation of the line passing through $(-8,-10)$ and parallel to the line whose equation is $y = -4x+3$. Express the equation in point-slope form and slope-intercept form.

Since the line is parallel to $y = 3x+1$, we know it will have slope $m = 3$.

We are given that it passes through $(-2,5)$. We use the slope and point to write the equation in point-slope form.
$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x - (-2))$$
$$y - 5 = 3(x+2)$$

Point-Slope form: $y - 5 = 3(x+2)$

Solve for y to obtain slope-intercept form.
$$y - 5 = 3(x+2)$$
$$y - 5 = 3x + 6$$
$$y = 3x + 11$$
$$f(x) = 3x + 11$$

Slope-Intercept form: $y = 3x + 11$

1b. Write an equation of the line passing through $(-2,-6)$ and perpendicular to the line whose equation is $x+3y-12=0$. Express the equation in point-slope form and general form.

First, find the slope of the line $x+3y-12=0$.

Solve the given equation for y to obtain slope-intercept form.

$$x+3y-12=0$$
$$3y=-x+12$$
$$y=-\frac{1}{3}x+4$$

Since the slope of the given line is $-\frac{1}{3}$, the slope of any line perpendicular to the given line is 3.

We use the slope of 3 and the point $(-2,-6)$ to write the equation in point-slope form. Then gather the variable and constant terms on one side with zero on the other side.

$$y-y_1=m(x-x_1)$$
$$y-(-6)=3(x-(-2))$$
$$y+6=3(x+2)$$
$$y+6=3x+6$$
$$0=3x-y \text{ or } 3x-y=0$$

1b. Write an equation of the line passing through $(4,-7)$ and perpendicular to the line whose equation is $x-2y-3=0$. Express the equation in point-slope form and general form.

Objective #2: Interpret slope as rate of change.

✔ *Solved Problem #2*

2. In 2000, there 11.9 million young adults, ages 25-34, living with parents and in 2017, there were 22.0 million young adults, ages 25-34, living with parents. Use the ordered pairs (2000, 11.9) and (2017, 22.0) to find the slope of the line through the points. Express the slope correct to two decimal places and describe what it represents.

$$m=\frac{\text{Change in } y}{\text{Change in } x}=\frac{22.0-11.9}{2017-2000}=\frac{10.1}{17}\approx0.59$$

The slope indicates that the percentage of U.S. young adults, age 25-34, that live with parents, increased at a rate of 0.59 per year. The rate of change is 0.59% per year.

Pencil Problem #2

2. In 1994, 617 active-duty servicemembers were discharged under the "don't ask, don't tell" policy. In 1998, 1163 were discharged under the policy. Use the ordered pairs (1994, 617) and (1998, 1163) to find the slope of the line through the points. Express the slope correct to the nearest whole number and describe what it represents.

Objective #3: Find a function's average rate of change..	
✔ **Solved Problem #3**	✎ **Pencil Problem #3**

3. Find the average rate of change of the function from x_1 to x_2.

3a. $f(x) = x^3$ from $x_1 = 0$ to $x_2 = 1$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(1) - f(0)}{1 - 0}$$
$$= \frac{1^3 - 0^3}{1}$$
$$= 1$$

The average rate of change is 1.

3b. $f(x) = x^3$ from $x_1 = 1$ to $x_2 = 2$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(2) - f(1)}{2 - 1}$$
$$= \frac{2^3 - 1^3}{1}$$
$$= \frac{8 - 1}{1}$$
$$= 7$$

The average rate of change is 7.

3c. $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(0) - f(-2)}{0 - (-2)}$$
$$= \frac{0^3 - (-2)^3}{0 + 2}$$
$$= \frac{0 - (-8)}{2}$$
$$= \frac{8}{2}$$
$$= 4$$

The average rate of change is 4.

3. Find the average rate of change of the function from x_1 to x_2.

3a. $f(x) = 3x$ from $x_1 = 0$ to $x_2 = 5$

3b. $f(x) = x^2 + 2x$ from $x_1 = 3$ to $x_2 = 5$

3c. $f(x) = \sqrt{x}$ from $x_1 = 4$ to $x_2 = 9$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. Point-Slope form: $y + 10 = -4(x + 8)$, Slope-Intercept form: $y = -4x - 42$ *(2.4 #5)*

1b. Point-Slope form: $y + 7 = -2(x - 4)$, General form: $2x + y - 1 = 0$ *(2.4 #11)*

2. 137; There was an average increase of approximately 137 discharges per year. *(2.4 #27)*

3a. 3 *(2.4 #13)*

3b. 10 *(2.4 #15)*

3c. $\dfrac{1}{5}$ *(2.4 #17)*

Section 2.5
Transformations of Functions

Movies and Mathematics

Have you ever seen special effects in a movie where a person or object is continuously transformed into something different? This is called morphing.

In mathematics, we can use transformations of a known graph to graph a function with a similar equation. This is achieved through horizontal and vertical shifts, reflections, and stretching and shrinking of the known graph.

Objective #1: Recognize graphs of common functions.

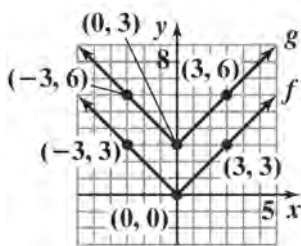| ✔ *Solved Problem #1*	✎ *Pencil Problem #1*
1. True or false: The graphs of the standard quadratic function $f(x) = x^2$ and the absolute value function $g(x) = \|x\|$ have the same type of symmetry.	1. True or false: The graphs of the identity function $f(x) = x$ and the standard cubic function $g(x) = x^3$ have the same type of symmetry.
True. Both functions are even and their graphs are symmetric with respect to the *y*-axis.	

Objective #2: Use vertical shifts to graph functions.

✔ *Solved Problem #2*	✎ *Pencil Problem #2*
2. Use the graph of $f(x) = \|x\|$ to obtain the graph of $g(x) = \|x\| + 3$.	2. Use the graph of $f(x) = x^2$ to obtain the graph of $g(x) = x^2 - 2$.

The graph of *g* is the graph of *f* shifted vertically up by 3 units. Add 3 to each *y*-coordinate.

Since the points (−3, 3), (0, 0), and (3, 3) are on the graph of *f*, the points (−3, 6), (0, 3), and (3, 6) are on the graph of *g*.
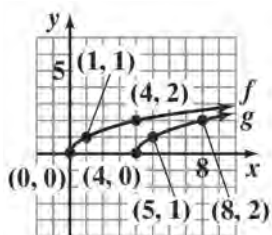

Objective #3: Use horizontal shifts to graph functions.

✔ Solved Problem #3

3. Use the graph of $f(x) = \sqrt{x}$ to obtain the graph of $g(x) = \sqrt{x-4}$.

$g(x) = \sqrt{x-4} = f(x-4)$

The graph of g is the graph of f shifted horizontally to the right by 4 units. Add 4 to each x-coordinate.

Since the points (0, 0), (1, 1), and (4, 2) are on the graph of f, the points (4, 0), (5, 1), and (8, 2) are on the graph of g.
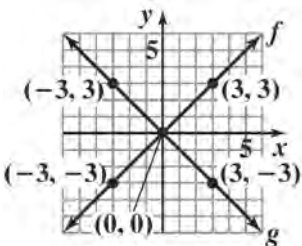

✎ Pencil Problem #3

3. Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = |x+4|$.

Objective #4: Use reflections to graph functions.

✔ Solved Problem #4

4a. Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = -|x|$.

The graph of g is a reflection of the graph of f about the x-axis because $g(x) = -f(x)$. Replace each y-coordinate with its opposite.

Since the points (−3, 3), (0, 0), and (3, 3) are on the graph of f, the points (−3, −3), (0, 0), and (3, −3) are on the graph of g.
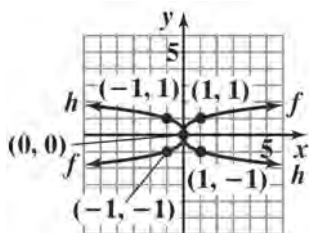

✎ Pencil Problem #4

4a. Use the graph of $f(x) = x^3$ to obtain the graph of $h(x) = -x^3$.
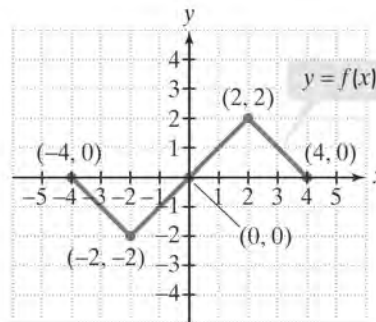

4b. Use the graph of $f(x) = \sqrt[3]{x}$ to obtain the graph of $h(x) = \sqrt[3]{-x}$.

The graph of *h* is a reflection of the graph of *f* about the *y*-axis because $h(x) = f(-x)$. Replace each *x*-coordinate with its opposite.

Since the points (−1, −1), (0, 0), and (1, 1) are on the graph of *f*, the points (1, −1), (0, 0), and (−1, 1) are on the graph of *h*.

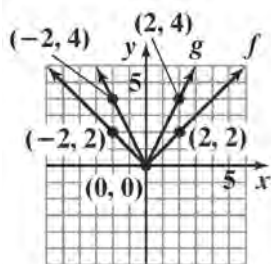**4b.** Use the graph of *f* shown below to obtain the graph of $g(x) = f(-x)$.

Objective #5: Use vertical stretching and shrinking to graph functions.

✔ ***Solved Problem #5***

5. Use the graph of $f(x) = |x|$ to obtain the graph of $g(x) = 2|x|$.

The graph of *g* is obtained by vertically stretching the graph of *f* because $g(x) = 2f(x)$. Multiply each *y*-coordinate by 2.

Since the points (−2, 2), (0, 0), and (2, 2) are on the graph of *f*, the points (−2, 4), (0, 0), and (2, 4) are on the graph of *g*.

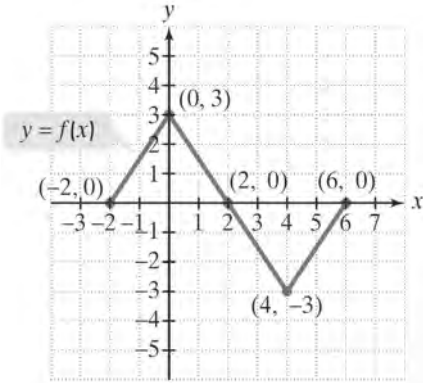✎ ***Pencil Problem #5***

5. Use the graph of $f(x) = x^3$ to obtain the graph of $h(x) = \dfrac{1}{2}x^3$.

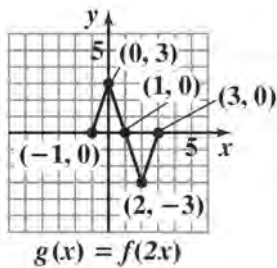**Objective #6:** Use horizontal stretching and shrinking to graph functions.

✔ *Solved Problem #6*	✏ *Pencil Problem #6*

6. Use the graph of *f* shown below to graph each function.

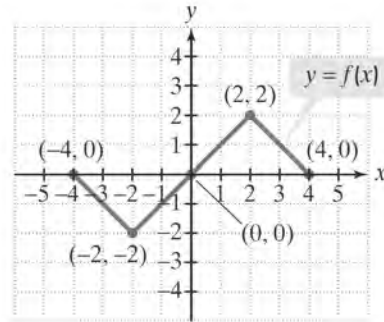**6.** Use the graph of *f* shown below to graph each function.

6a. $g(x) = f(2x)$

Divide the *x*-coordinate of each point on the graph of *f* by 2. The points (−1, 0), (0, 3), (1, 0), (2, −3), and (3, 0) are on the graph of *g*.

6a. $g(x) = f(2x)$

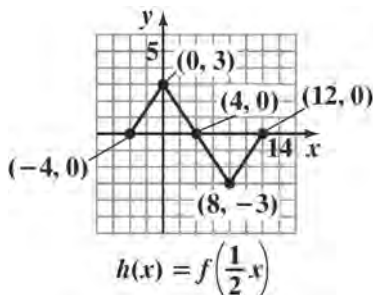---

6b. $h(x) = f\left(\frac{1}{2}x\right)$

Multiply the *x*-coordinate of each point on the graph of *f* by 2. The points (−4, 0), (0, 3), (4, 0), (8, −3), and (12, 0) are on the graph of *h*.

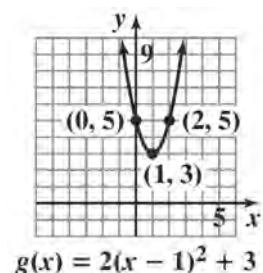**6b.** $g(x) = f\left(\frac{1}{2}x\right)$

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Objective #7: Graph functions involving a sequence of transformations.

✔ *Solved Problem #7*	🖋 *Pencil Problem #7* 🖋
7. Use the graph of $f(x) = x^2$ to graph $g(x) = 2(x-1)^2 + 3$.	**7.** Use the graph of $f(x) = x^3$ to graph $h(x) = \frac{1}{2}(x-3)^2 - 2$.

The graph of g is the graph of f horizontally shifted to the right 1 unit, vertically stretched by a factor of 2, and vertically shifted up 3 units. Beginning with a point on the graph of f, add 1 to each x-coordinate, then multiply each y-coordinate by 2, and finally add 3 to each y-coordinate.

$(-1, 1) \to (0, 1) \to (0, 2) \to (0, 5)$
$(0, 0) \to (1, 0) \to (1, 0) \to (1, 3)$
$(1, 1) \to (2, 1) \to (2, 2) \to (2, 5)$

$g(x) = 2(x - 1)^2 + 3$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. True   *(2.5 #99)*
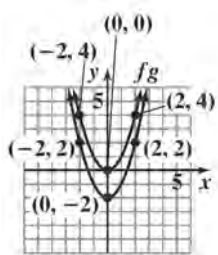
2.   *(2.5 #53)*

3.   *(2.5 #83)*

4a.   *(2.5 #99)*

4b.
$g(x) = f(-x)$ *(2.5 #24)*

5.   *(2.5 #101)*

6a.   *(2.5 #29)*
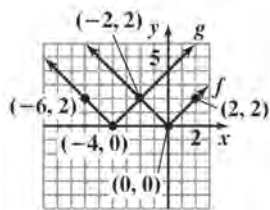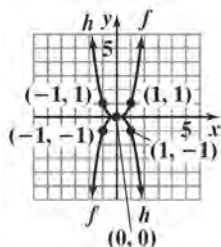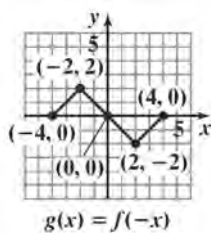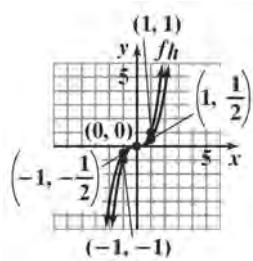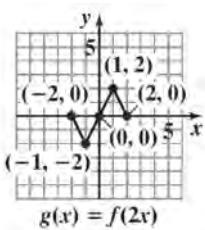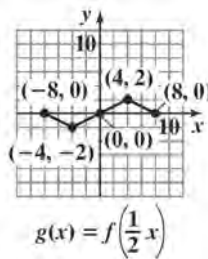
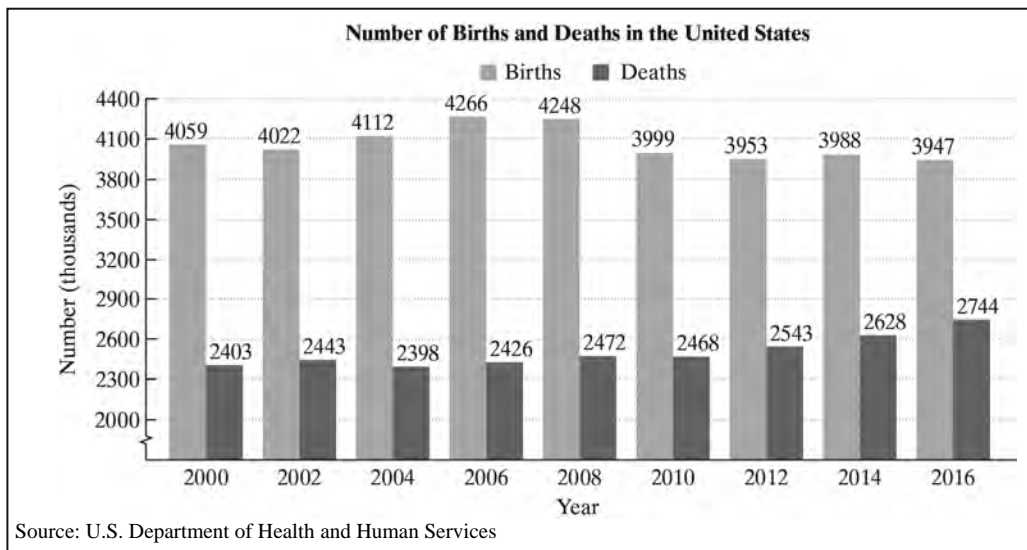**6b.**   *(2.5 #30)*
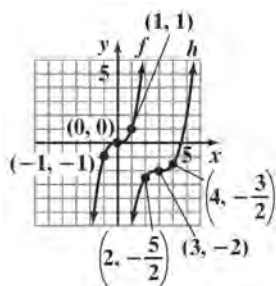
7.   *(2.5 #105)*

Section 2.6
Combinations of Functions; Composite Functions

We're Born. We Die.

The figure below quantifies these statements by showing the number of births and deaths in the United States from 2000 through 2016.

Number of Births and Deaths in the United States

Source: U.S. Department of Health and Human Services

In this section, we look at these data from the perspective of functions. By considering the yearly change in the U.S. population, you will see that functions can be subtracted using procedures that will remind you of combining algebraic expressions.

Objective #1: Find the domain of a function.

✔ *Solved Problem #1*	✎ *Pencil Problem #1*
1a. Find the domain of $f(x) = x^2 + 3x - 17$.	**1a.** Find the domain of $f(x) = 3(x-4)$.
The function contains neither division nor an even root. It is defined for all real numbers. The domain is $(-\infty, \infty)$.	

1b. Find the domain of $j(x) = \dfrac{5x}{\sqrt{24-3x}}$.	**1b.** Find the domain of $g(x) = \dfrac{\sqrt{x-2}}{x-5}$.
The function contains both an even root and division. The expression under the radical must be nonnegative and the denominator cannot equal 0. Thus, $24 - 3x$ must be greater than 0.	

$24 - 3x > 0$

$24 > 3x$

$8 > x$ or $x < 8$

The domain is $(-\infty, 8)$.

Objective #2: Combine functions using the algebra of functions, specifying domains.	
✔ *Solved Problem #2*	✎ *Pencil Problem #2*

2. Let $f(x) = x - 5$ and $g(x) = x^2 - 1$. Find each function and determine its domain.

2. Let $f(x) = 2x^2 - x - 3$ and $g(x) = x + 1$. Find each function and determine its domain.

2a. $(f + g)(x)$

$$(f + g)(x) = f(x) + g(x)$$
$$= (x - 5) + (x^2 - 1)$$
$$= x^2 + x - 6$$

The domain is $(-\infty, \infty)$.

2a. $(f + g)(x)$

2b. $(f - g)(x)$

$$(f - g)(x) = f(x) - g(x)$$
$$= (x - 5) - (x^2 - 1)$$
$$= x - 5 - x^2 + 1$$
$$= -x^2 + x - 4$$

The domain is $(-\infty, \infty)$.

2b. $(f - g)(x)$

2c. $(fg)(x)$

$$(fg)(x) = f(x) \cdot g(x)$$
$$= (x - 5)(x^2 - 1)$$
$$= x^3 - 5x^2 - x + 5$$

The domain is $(-\infty, \infty)$.

2c. $(fg)(x)$

2d. $\left(\dfrac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x - 5}{x^2 - 1}$$

The function contains division; it is undefined when $x^2 - 1 = 0$ or $x^2 = 1$ or $x = \pm 1$. The domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

2d. $\left(\dfrac{f}{g}\right)(x)$

Objective #3: Form composite functions.	
✔ *Solved Problem #3*	✎ *Pencil Problem #3*

3. Given $f(x) = 5x + 6$ and $g(x) = 2x^2 - x - 1$, find each of the following.

3a. $(f \circ g)(x)$

$(f \circ g)(x) = f(g(x))$
$= 5g(x) + 6$
$= 5(2x^2 - x - 1) + 6$
$= 10x^2 - 5x - 5 + 6$
$= 10x^2 - 5x + 1$

3. Given $f(x) = 4x - 3$ and $g(x) = 5x^2 - 2$, find each of the following.

3a. $(f \circ g)(x)$

3b. $(g \circ f)(x)$

$(g \circ f)(x) = g(f(x))$
$= 2[f(x)]^2 - f(x) - 1$
$= 2(5x + 6)^2 - (5x + 6) - 1$
$= 2(25x^2 + 60x + 36) - 5x - 6 - 1$
$= 50x^2 + 120x + 72 - 5x - 7$
$= 50x^2 + 115x + 65$

3b. $(g \circ f)(x)$

3c. $(f \circ g)(-1)$

$(f \circ g)(-1) = 10(-1)^2 - 5(-1) + 1$
$= 10 + 5 + 1$
$= 16$

3c. $(f \circ g)(2)$

Objective #4: Determine domains for composite functions.	
✔ *Solved Problem #4*	✎ *Pencil Problem #4*

4. Given $f(x) = \dfrac{4}{x+2}$ and $g(x) = \dfrac{1}{x}$, find each of the following.

4a. $(f \circ g)(x)$

$(f \circ g)(x) = \dfrac{4}{g(x)+2} = \dfrac{4}{\frac{1}{x}+2} \cdot \dfrac{x}{x} = \dfrac{4x}{1+2x}$

4. Given $f(x) = \dfrac{2}{x+3}$ and $g(x) = \dfrac{1}{x}$, find each of the following.

4a. $(f \circ g)(x)$

4b. The domain of $f \circ g$

4b. The domain of $f \circ g$

The function g is undefined when $x = 0$, so 0 is not in the domain of $f \circ g$. The function f is undefined for $x = -2$, so any values of x for which $g(x) = -2$ are not in the domain of $f \circ g$. Solving $\dfrac{1}{x} = -2$, we find that $x = -\dfrac{1}{2}$.

The domain is $\left(-\infty, -\dfrac{1}{2}\right) \cup \left(-\dfrac{1}{2}, 0\right) \cup (0, \infty)$.

Objective #5: Write functions as compositions.

✔ Solved Problem #5

5. Express $h(x) = \sqrt{x^2 + 5}$ as the composition of two functions.

A natural way to write h as the composition of two functions is to take the square root of $g(x) = x^2 + 5$.

Let $f(x) = \sqrt{x}$ and $g(x) = x^2 + 5$.

Then $(f \circ g)(x) = \sqrt{g(x)} = \sqrt{x^2 + 5} = h(x)$.

✎ Pencil Problem #5

5. Express $h(x) = (3x - 1)^4$ as the composition of two functions.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $(-\infty, \infty)$ *(2.6 #1)*   **1b.** $[2, 5) \cup (5, \infty)$ *(2.6 #27)*

2a. $(f + g)(x) = 2x^2 - 2$; domain: $(-\infty, \infty)$ *(2.6 #35)*

2b. $(f - g)(x) = 2x^2 - 2x - 4$; domain: $(-\infty, \infty)$ *(2.6 #35)*

2c. $(fg)(x) = 2x^3 + x^2 - 4x - 3$; domain: $(-\infty, \infty)$ *(2.6 #35)*

2d. $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x^2 - x - 3}{x + 1}$; domain: $(-\infty, -1) \cup (1, \infty)$ *(2.6 #35)*

3a. $(f \circ g)(x) = 20x^2 - 11$ *(2.6 #55a)*   **3b.** $(g \circ f)(x) = 80x^2 - 120x + 43$ *(2.6 #55 b)*   **3c.** 69 *(2.6 #55c)*

4a. $(f \circ g)(x) = \dfrac{2x}{1 + 3x}$ *(2.6 #67a)*   **4b.** $\left(-\infty, -\dfrac{1}{3}\right) \cup \left(-\dfrac{1}{3}, 0\right) \cup (0, \infty)$ *(2.6 #67b)*

5. Let $f(x) = x^4$ and $g(x) = 3x - 1$. Then $(f \circ g)(x) = h(x)$. *(2.6 #75)*

Section 2.7
Inverse Functions

Hey! That's My Birthday Too !

What is the probability that two people in the same room share a birthday?

It might be higher than you think.

In this section we will explore the graph of the function that represents this probability.

Objective #1: Verify inverse functions.

✔ *Solved Problem #1*

1. Show that each function is the inverse of the other:

    $f(x) = 4x - 7$ and $g(x) = \dfrac{x+7}{4}$.

First, show that $f(g(x)) = x$.

$f(g(x)) = 4\left(\dfrac{x+7}{4}\right) - 7$

$\qquad = x + 7 - 7$

$\qquad = x$

Next, show that $g(f(x)) = x$.

$g(f(x)) = \dfrac{(4x-7)+7}{4}$

$\qquad = \dfrac{4x}{4}$

$\qquad = x$

✎ *Pencil Problem #1*

1. Determine whether $f(x) = \dfrac{3}{x-4}$ and $g(x) = \dfrac{3}{x} + 4$

    are inverses of each other.

Objective #2: Find the inverse of a function.

✔ *Solved Problem #2*

2a. Find the inverse of $f(x) = 2x + 7$.

Replace $f(x)$ with y.

$y = 2x + 7$

Interchange x and y and solve for y.

$\quad x = 2y + 7$

$x - 7 = 2y$

$\dfrac{x-7}{2} = y$

Replace y with $f^{-1}(x)$.

$f^{-1}(x) = \dfrac{x-7}{2}$

✎ *Pencil Problem #2*

2a. Find the inverse of $f(x) = x + 3$.

2b. Find the inverse of $f(x) = 4x^3 - 1$.

Replace $f(x)$ with y.
$y = 4x^3 - 1$

Interchange x and y and solve for y.
$$x = 4y^3 - 1$$
$$x + 1 = 4y^3$$
$$\frac{x+1}{4} = y^3$$
$$\sqrt[3]{\frac{x+1}{4}} = y$$

Replace y with $f^{-1}(x)$.
$$f^{-1}(x) = \sqrt[3]{\frac{x+1}{4}}$$

2b. Find the inverse of $f(x) = (x+2)^3$.

2c. Find the inverse of $f(x) = \dfrac{x+1}{x-5}$.

Replace $f(x)$ with y.
$$y = \frac{x+1}{x-5}$$

Interchange x and y and solve for y.
$$x = \frac{y+1}{y-5}$$
$$x(y-5) = y+1$$
$$xy - 5x = y+1$$
$$xy - y = 5x + 1$$
$$y(x-1) = 5x + 1$$
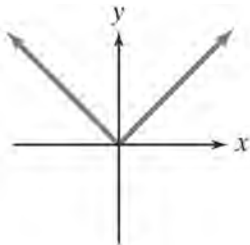$$y = \frac{5x+1}{x-1}$$

Replace y with $f^{-1}(x)$.
$$f^{-1}(x) = \frac{5x+1}{x-1}$$

2c. Find the inverse of $f(x) = \dfrac{x+4}{x-2}$.

Objective #3: Use the horizontal line test to determine if a function has an inverse function.

✔ Solved Problem #3

3. Use the horizontal line test to determine if the following graph represents a function that has an inverse function.
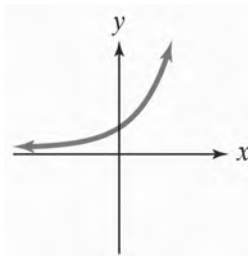
Since a horizontal line can be drawn that intersects the graph more than once, it fails the horizontal line test.

Thus, this graph does not represent a function that has an inverse function.

✎ Pencil Problem #3✎

3. Use the horizontal line test to determine if the following graph represents a function that has an inverse function.

Objective #4: Use the graph of a one-to-one function to graph its inverse function.

✔ Solved Problem #4

4. The graph of function f consists of two line segments, one segment from $(-2,-2)$ to $(-1,0)$, and a second segment from $(-1,0)$ to $(1,2)$. Graph f and use the graph to draw the graph of its inverse function.

Since f has a line segment from $(-2,-2)$ to $(-1,0)$, then f^{-1} has a line segment from $(-2,-2)$ to $(0,-1)$.

Since f has a line segment from $(-1,0)$ to $(1,2)$, then f^{-1} has a line segment from $(0,-1)$ to $(2,1)$.
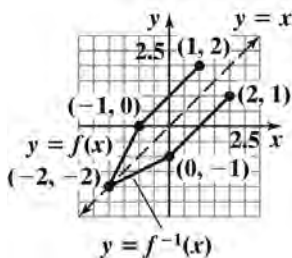
✎ Pencil Problem #4✎

4. The graph of a linear function f contains the points $(0,-4)$, $(2,0)$, $(3,2)$, and $(4,4)$. Draw the graph of the inverse function.

Objective #5: Find the inverse of a function and graph both functions on the same axes.

| ✔ *Solved Problem #5* | ✎ *Pencil Problem #5* |

5. Find the inverse of $f(x) = x^2 + 1$ if $x \geq 0$. Graph f and f^{-1} in the same rectangular coordinate system.

Restricted to $x \geq 0$, the function $f(x) = x^2 + 1$ has an inverse. The graph of f is the right half of the graph of $y = x^2$ shifted up 1 unit.

Replace $f(x)$ with y: $y = x^2 + 1$.

Interchange x and y and solve for y. Since the values of x are nonnegative in the original function, the values of y must be nonnegative in the inverse function. We choose the positive square root in the third step below.
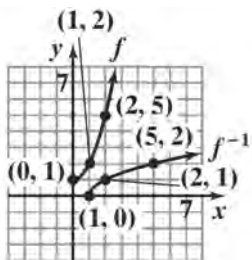
$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\sqrt{x-1} = y$$

Replace y with $f^{-1}(x)$: $f^{-1}(x) = \sqrt{x-1}$.

The graph of f^{-1} is the graph of the square root function shifted 1 unit to the left. The graph of f^{-1} is also the reflection of the graph of f about the line $y = x$.
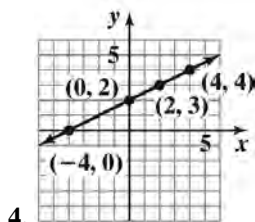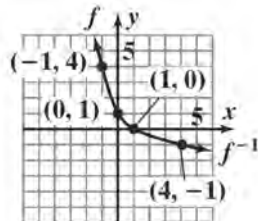

5. Find the inverse of $f(x) = (x-1)^2$ if $x \leq 1$. Graph f and f^{-1} in the same rectangular coordinate system.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. The functions are inverses of each other. *(2.7 #7)*   **2a.** $f^{-1}(x) = x - 3$ *(2.7 #11)*

2b. $f^{-1}(x) = \sqrt[3]{x} - 2$ *(2.7 #19)*   **2c.** $f^{-1}(x) = \dfrac{2x+4}{x-1}$ *(2.7 #25)*   **3.** has inverse function *(2.7 #33)*

4.   *(2.7 #35)*   **5.**   *(2.7 #43)*

Section 2.8
Distance and Midpoint Formulas; Circles

Round and Round !

In 1893, George Washington Gale Ferris, Jr. designed and built the first Ferris wheel as the centerpiece for the World's Columbian Exposition in Chicago.

The rectangular coordinate system gives us a unique way of knowing a circle. It enables us to translate a circle's geometric definition into an algebraic equation. In this section, we will learn, and then apply, these algebraic techniques.

Objective #1: Find the distance between two points.

✔ Solved Problem #1

1. Find the distance between $(-1,-3)$ and $(2,3)$.

   Express the answer in simplified radical form and then round to two decimal places.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(2-(-1))^2 + (3-(-3))^2}$$
$$= \sqrt{3^2 + 6^2}$$
$$= \sqrt{9 + 36}$$
$$= \sqrt{45}$$
$$= 3\sqrt{5}$$
$$\approx 6.71 \text{ units}$$

✎ Pencil Problem #1 ✏

1. Find the distance between $(4,-1)$ and $(-6,3)$.

   Express the answer in simplified radical form and then round to two decimal places.

Objective #2: Find the midpoint of a line segment.

✔ Solved Problem #2

2. Find the midpoint of the line segment with endpoints $(1,2)$ and $(7,-3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$
$$= \left(\frac{1+7}{2}, \frac{2+(-3)}{2} \right)$$
$$= \left(\frac{8}{2}, \frac{-1}{2} \right)$$
$$= \left(4, -\frac{1}{2} \right)$$

✎ Pencil Problem #2 ✏

2. Find the midpoint of the line segment with endpoints $(6,8)$ and $(2,4)$.

**Objective #3:** Write the standard form of a circle's equation.

✔ *Solved Problem #3* | ✎ *Pencil Problem #3* ✎

3a. Write the standard form of the equation of the circle with center $(0,0)$ and radius 4.

$(x-h)^2 + (y-k)^2 = r^2$
$(x-0)^2 + (y-0)^2 = 4^2$
$x^2 + y^2 = 16$

3a. Write the standard form of the equation of the circle with center $(0,0)$ and radius 7.

3b. Write the standard form of the equation of the circle with center $(0,-6)$ and radius 10.

$(x-h)^2 + (y-k)^2 = r^2$
$(x-0)^2 + (y-(-6))^2 = 10^2$
$x^2 + (y+6)^2 = 100$

3b. Write the standard form of the equation of the circle with center $(-1,4)$ and radius 2.

Objective #4: Give the center and radius of a circle whose equation is in standard form.

✔ *Solved Problem #4* | ✎ *Pencil Problem #4* ✎

4a. Find the center and radius of the circle whose equation is $(x+3)^2 + (y-1)^2 = 4$.

$(x+3)^2 + (y-1)^2 = 4$
$(x-(-3))^2 + (y-1)^2 = 2^2$

The center is $(-3,1)$ and the radius is 2 units.

4a. Find the center and radius of the circle whose equation is $(x-3)^2 + (y-1)^2 = 36$.

4b. Graph the equation in Solved Problem 4a.

Plot points 2 units above and below and to the left and right of the center, $(-3,1)$. Draw a circle through these points.

$(x+3)^2 + (y-1)^2 = 4$
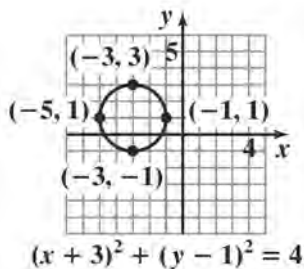
4b. Graph the equation in Pencil Problem 4a.

4c. Use the graph in Solved Problem 4b to identify the relation's domain and range.

The leftmost point on the circle has an x-coordinate of -5, and the rightmost point has an x-coordinate of -1. The domain is $[-5, -1]$.

The lowest point on the graph has a y-coordinate of -1, and the highest point on the graph has a y-coordinate of 3. The range is $[-1, 3]$.

4c. Use the graph in Pencil Problem 4b to identify the relation's domain and range.

Objective #5: Convert the general form of a circle's equation to standard form.

✔ *Solved Problem #5*

5. Write in standard form and graph:
$$x^2 + y^2 + 4x - 4y - 1 = 0$$

$$x^2 + y^2 + 4x - 4y - 1 = 0$$
$$\left(x^2 + 4x \quad\right) + \left(y^2 - 4y \quad\right) = 1$$

Complete the squares.

For x: $\left(\dfrac{b}{2}\right)^2 = \left(\dfrac{4}{2}\right)^2 = (2)^2 = 4$

For y: $\left(\dfrac{b}{2}\right)^2 = \left(\dfrac{-4}{2}\right)^2 = (-2)^2 = 4$

Pencil Problem #5

5. Write in standard form and graph:
$$x^2 + y^2 + 8x - 2y - 8 = 0$$

Add these values to both sides of the equation.
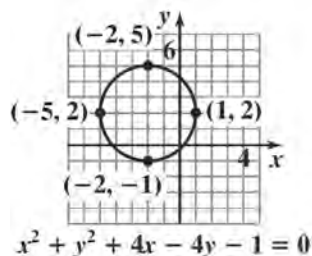
$$x^2 + y^2 + 4x - 4y - 1 = 0$$
$$\left(x^2 + 4x \quad\right) + \left(y^2 - 4y \quad\right) = 1$$
$$\left(x^2 + 4x + 4\right) + \left(y^2 - 4y + 4\right) = 1 + 4 + 4$$
$$(x+2)^2 + (y-2)^2 = 9$$
$$(x-(-2))^2 + (y-2)^2 = 3^2$$
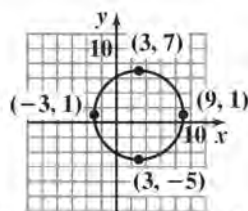
The center is $(-2, 2)$ and the radius is 3 units.

$x^2 + y^2 + 4x - 4y - 1 = 0$

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1. $2\sqrt{29} \approx 10.77$ *(2.8 #3)*   **2.** $(4, 6)$ *(2.8 #19)*

3a. $x^2 + y^2 = 49$ *(2.8 #31)*   **3b.** $(x+1)^2 + (y-4)^2 = 4$ *(2.8 #35)*
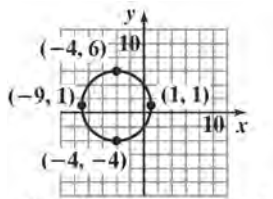

4a. center: $(3, 1)$; radius: 6 units   *(2.8 #43)*   **4b.** $(x-3)^2 + (y-1)^2 = 36$ *(2.8 #43)*

4c. domain: $[-3, 9]$; range: $[-5, 7]$ *(2.8 #43)*

5. $(x+4)^2 + (y-1)^2 = 25$;

$x^2 + y^2 + 8x - 2y - 8 = 0$ *(2.8 #57)*