Section 1.1
Graphs and Graphing Utilities

Let it snow! Let it snow! Let it snow!

The arrival of snow can range from light flurries to a full-fledged blizzard. Snow can be welcomed as a beautiful backdrop to outdoor activities or it can be a nuisance and endanger drivers.

We will look at how graphs can be used to explain both mathematical concepts and everyday situations. Specifically, in the application exercises of this section of the textbook, you will match stories of varying snowfalls to the graphs that explain them.

Objective #1: Plot points in the rectangular coordinate system.

✔ *Solved Problem #1*

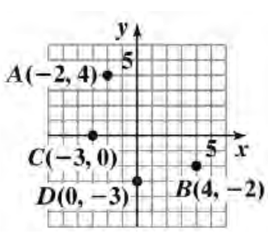
1a. Plot the points:
A(−2, 4), *B*(4, −2), *C*(−3, 0), and *D*(0, −3).

From the origin, point *A* is left 2 units and up 4 units.

From the origin, point *B* is right 4 units and down 2 units.

From the origin, point *C* is left 3 units.

From the origin, point *D* is down 3 units.



1b. If a point is on the *x*-axis it is neither up nor down, so $x = 0$.

False. The *y*-coordinate gives the distance up or down, so $y = 0$.

Pencil Problem #1

1a. Plot the points:
A(1, 4), *B*(−2, 3), *C*(−3, −5), and *D*(−4, 0).

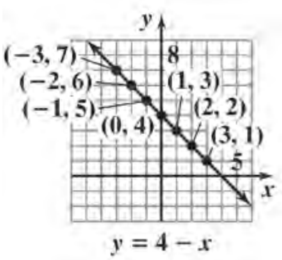
1b. True or false: If a point is on the *y*-axis, its *x*-coordinate must be 0.

Objective #2: Graph equations in the rectangular coordinate system.

✔ *Solved Problem #2*

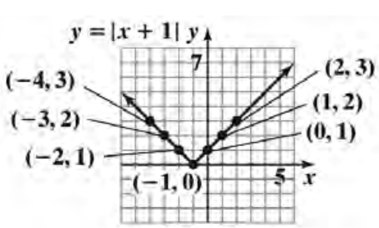
2a. Graph $y = 4 - x$.

x	$y = 4 - x$	(x, y)
-3	$y = 4 - (-3) = 7$	$(-3, 7)$
-2	$y = 4 - (-2) = 6$	$(-2, 6)$
-1	$y = 4 - (-1) = 5$	$(-1, 5)$
0	$y = 4 - (0) = 4$	$(0, 4)$
1	$y = 4 - (1) = 3$	$(1, 3)$
2	$y = 4 - (2) = 2$	$(2, 2)$
3	$y = 4 - (3) = 1$	$(3, 1)$



2b. Graph $y = |x + 1|$.

x	$y = \|x+1\|$	(x, y)
-4	$y = \|-4+1\| = \|-3\| = 3$	$(-4, 3)$
-3	$y = \|-3+1\| = \|-2\| = 2$	$(-3, 2)$
-2	$y = \|-2+1\| = \|-1\| = 1$	$(-2, 1)$
-1	$y = \|-1+1\| = \|0\| = 0$	$(-1, 0)$
0	$y = \|0+1\| = \|1\| = 1$	$(0, 1)$
1	$y = \|1+1\| = \|2\| = 2$	$(1, 2)$
2	$y = \|2+1\| = \|3\| = 3$	$(2, 3)$



Pencil Problem #2

2a. Graph $y = x^2 - 2$. Let $x = -3, -2, -1, 0, 1, 2,$ and 3.

2b. Graph $y = 2|x|$. Let $x = -3, -2, -1, 0, 1, 2,$ and 3.

Objective #3: Interpret information about a graphing utility's viewing rectangle or table.

✔ ***Solved Problem #3***

3. What is the meaning of a [–100, 100, 50] by [–100, 100, 10] viewing rectangle?

The minimum *x*-value is –100, the maximum *x*-value is 100, and the distance between consecutive tick marks is 50.

The minimum *y*-value is –100, the maximum *y*-value is 100, and the distance between consecutive tick marks is 10.

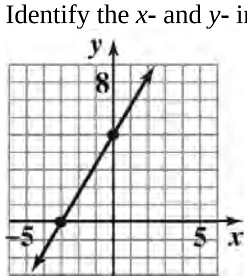
Pencil Problem #3

3. What is the meaning of a [–20, 80, 10] by [–30, 70, 10] viewing rectangle?

Objective #4: Use a graph to determine intercepts.

✔ ***Solved Problem #4***

4a. Identify the *x*- and *y*- intercepts:

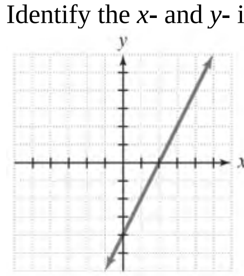


The graph crosses the *x*-axis at (–3,0). Thus, the *x*-intercept is –3.

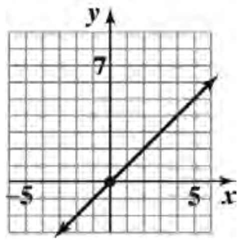
The graph crosses the *y*-axis at (0,5). Thus, the *y*-intercept is 5.

Pencil Problem #4

4a. Identify the *x*- and *y*- intercepts:



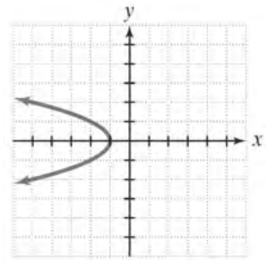
4b. Identify the *x*- and *y*- intercepts:



The graph crosses the *x*-axis at (0,0).
Thus, the *x*-intercept is 0.

The graph crosses the *y*-axis at (0,0).
Thus, the *y*-intercept is 0.

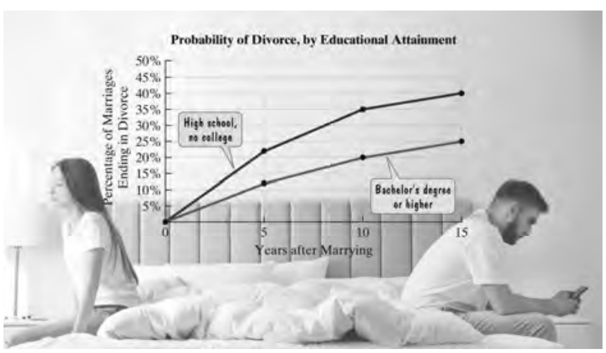
4b. Identify the *x*- and *y*- intercepts:



Objective #5: Interpret information given by graphs.

✔ ***Solved Problem #5***

5. The line graphs show the percentage of marriages ending in divorce based two levels of educational attainment.



The model $d = 1.8n + 14$ approximates the data in the graph for high school graduates with no college. In the model, *n* is the number of years after marriage and *d* is the percentage of marriages ending in divorce.

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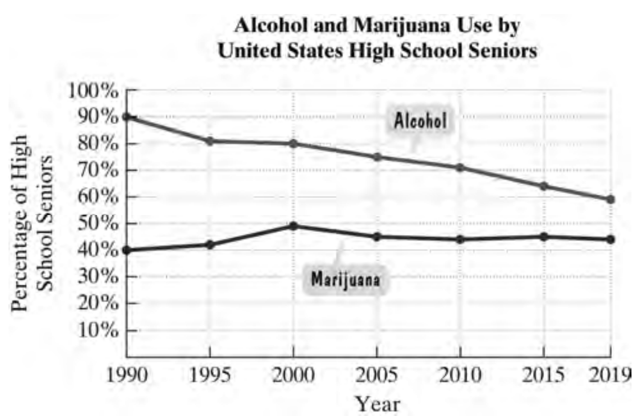
Pencil Problem #5

5. The graphs show the percentage of high school seniors who used alcohol or marijuana.



The data for seniors who used marijuana can be modeled by $M = 0.1n + 43$, where *M* is the percentage of seniors who used marijuana *n* years after 1990.

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5a. Use the formula to determine the percentage of marriages ending in divorce after 15 years for high school graduates with no college.

$$d = 1.8n + 14$$
$$d = 1.8(15) + 14 = 41$$

According to the formula, 41% of marriages end in divorce after 15 years for high school graduates with no college.

5a. Use the formula to determine the percentage of seniors who used marijuana in 2010.

5b. Use the appropriate line graph to determine the percentage of marriages ending in divorce after 15 years for high school graduates with no college.

Locate 15 on the horizontal axis and locate the point above it on the graph. Read across to the corresponding percentage on the vertical axis. This percentage is 40. According to the line graph, 40% of marriages end in divorce after 15 years for high school graduates with no college.

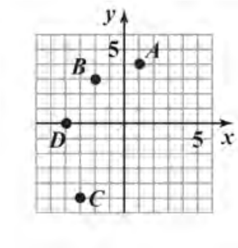
5b. Use the appropriate line graph to determine the percentage of seniors who used marijuana in 2010.

5c. Does the value given by the model underestimate or overestimate the value shown by the graph? By how much?

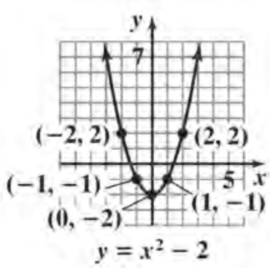
The value given by the model, 41%, is greater than the value shown by the graph, 40%, so the model overestimates the percentage by $41 - 40$, or 1.

5c. Does the formula underestimate or overestimate the percentage of seniors who used marijuana in 2010 as shown by the graph.

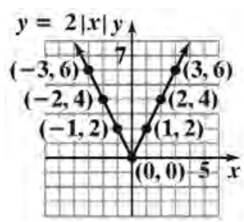
Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:



1a. *(1.1 #1-9)* **1b.** True *(1.1.#73)*



2a. *(1.1 #13)* **2b.**



(1.1 #21)

3. The minimum x-value is –20, the maximum x-value is 80, and the distance between consecutive tick marks is 10. The minimum y-value is –30, the maximum y-value is 70, and the distance between consecutive tick marks is 10. *(1.1 #31)*

4a. x-intercept: 2; y-intercept: -4 *(1.1 #41)* **4b.** x-intercept: -1 ; y-intercept: none *(1.1 #45)*

5a. 45% *(1.1 #55b)* **5b.** ≈44% *(1.1 #55a)* **5c.** overestimates by 1, although answers vary *(1.1 #55b)*

Section 1.2
Linear Equations and Rational Equations

Up, Up, and Away!

Inflation!
It seems that everything costs more and more each year.

What cost $10,000 in 1967 would have cost you $51,100 in 1999 and $79,100 in 2020!

In the Exercise Set of this section of the textbook, we will look at mathematical formulas that model this increase.

Objective #1: Solve linear equations in one variable

✔ *Solved Problem #1*

1a. Solve and check: $4x+5=29$

$$4x+5=29$$
$$4x+5-5=29-5$$
$$4x=24$$
$$\frac{4x}{4}=\frac{24}{4}$$
$$x=6$$

Check:
$$4x+5=29$$
$$4(6)+5=29$$
$$24+5=29$$
$$29=29$$

The check verifies that the solution set is $\{6\}$.

1b. Solve and check: $4(2x+1)=29+3(2x-5)$

Simplify the algebraic expression on each side.
$$4(2x+1)=29+3(2x-5)$$
$$8x+4=29+6x-15$$
$$8x+4=6x+14$$

Collect variable terms on one side and constant terms on the other side.
$$8x-6x+4=6x-6x+14$$
$$2x+4=14$$
$$2x+4=14$$
$$2x+4-4=14-4$$

Pencil Problem #1

1a. Solve and check: $6x-3=63$

1b. Solve and check: $16=3(x-1)-(x-7)$

Isolate the variable and solve.

$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

Check:

$$4(2x+1) = 29 + 3(2x-5)$$
$$4(2 \cdot 5 + 1) = 29 + 3(2 \cdot 5 - 5)$$
$$4(11) = 29 + 3(5)$$
$$44 = 44$$

The solution set is $\{5\}$.

Objective #2: Solve linear equations containing fractions.

✔ *Solved Problem #2*

2. Solve and check: $\frac{x-3}{4} = \frac{5}{14} - \frac{x+5}{7}$

The LCD is 28.

$$\frac{x-3}{4} = \frac{5}{14} - \frac{x+5}{7}$$
$$28\left(\frac{x-3}{4}\right) = 28\left(\frac{5}{14} - \frac{x+5}{7}\right)$$
$$\frac{28}{1}\left(\frac{x-3}{4}\right) = \frac{28}{1}\left(\frac{5}{14}\right) - \frac{28}{1}\left(\frac{x+5}{7}\right)$$
$$7(x-3) = 2(5) - 4(x+5)$$
$$7x - 21 = 10 - 4x - 20$$
$$7x - 21 = -4x - 10$$
$$7x + 4x - 21 = -4x + 4x - 10$$
$$11x - 21 = -10$$
$$11x - 21 + 21 = -10 + 21$$
$$11x = 11$$
$$\frac{11x}{11} = \frac{11}{11}$$
$$x = 1$$

Check:

$$\frac{x-3}{4} = \frac{5}{14} - \frac{x+5}{7}$$
$$\frac{1-3}{4} = \frac{5}{14} - \frac{1+5}{7}$$
$$\frac{-2}{4} = \frac{5}{14} - \frac{6}{7}$$
$$-\frac{1}{2} = -\frac{1}{2}$$

The solution set is $\{1\}$.

Pencil Problem #2

2. Solve and check: $\frac{x+3}{6} = \frac{3}{8} + \frac{x-5}{4}$

Objective #3: Solve rational equations with variables in denominators.

Solved Problem #3

3a. Solve: $\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x}$

The LCD is 18*x*. Two of the denominators would equal 0 if $x = 0$, so $x \neq 0$.

$$\frac{5}{2x}=\frac{17}{18}-\frac{1}{3x},\ x\neq 0$$
$$18x\cdot\frac{5}{2x}=18x\cdot\left(\frac{17}{18}-\frac{1}{3x}\right)$$
$$18x\cdot\frac{5}{2x}=18x\cdot\frac{17}{18}-18x\cdot\frac{1}{3x}$$
$$9\cdot 5=x\cdot 17-6\cdot 1$$
$$45=17x-6$$
$$45+6=17x-6+6$$
$$51=17x$$
$$\frac{51}{17}=\frac{17x}{17}$$
$$3=x$$

Note that 3 is not part of the restriction $x \neq 0$. The solution set is $\{3\}$.

Pencil Problem #3

3a. Solve: $\frac{4}{x}=\frac{5}{2x}+3$

3b. Solve: $\frac{x}{x-2}=\frac{3}{x-2}-\frac{2}{3}$

The LCD is $3(x-2)$. Two of the denominators would equal 0 if $x = 2$, so $x \neq 2$.

$$\frac{x}{x-2}=\frac{2}{x-2}-\frac{2}{3},\ x\neq 2$$
$$3(x-2)\cdot\frac{x}{x-2}=3(x-2)\cdot\left(\frac{2}{x-2}-\frac{2}{3}\right)$$
$$3(x-2)\cdot\frac{x}{x-2}=3(x-2)\cdot\frac{2}{x-2}-3(x-2)\cdot\frac{2}{3}$$
$$3x=3\cdot 2-(x-2)\cdot 2$$
$$3x=6-2x+4$$
$$3x=10-2x$$
$$3x+2x=10-2x+2x$$
$$5x=10$$
$$\frac{5x}{5}=\frac{10}{5}$$
$$x=2$$

The proposed solution is not a solution because of the restriction $x \neq 2$. The solution set is $\varnothing$.

3b. Solve: $\frac{8x}{x+1}=4-\frac{8}{x+1}$

Objective #4: Recognize identities, conditional equations, and inconsistent equations.

✔ Solved Problem #4

4a. Solve and determine whether the equation is an identity, a conditional equation, or an inconsistent equation.
$4x-7=4(x-1)+3$

$$\begin{aligned}4x-7&=4(x-1)+3\\4x-7&=4x-4+3\\4x-7&=4x-1\\-7&=-1\end{aligned}$$

This equation is an inconsistent equation and thus has no solution.

The solution set is { } or $\varnothing$.

4b. Solve and determine whether the equation is an identity, a conditional equation, or an inconsistent equation.
$7x+9=9(x+1)-2x$

$$\begin{aligned}7x+9&=9(x+1)-2x\\7x+9&=9x+9-2x\\7x+9&=7x+9\\9&=9\end{aligned}$$

This equation is an identity and all real numbers are solutions.

The solution set is $\{x \mid x \text{ is a real number}\}$ or $(-\infty,\infty)$ or $\mathbb{R}$.

Pencil Problem #4

4a. Solve and determine whether the equation is an identity, a conditional equation, or an inconsistent equation.
$5x+9=9(x+1)-4x$

4b. Solve and determine whether the equation is an identity, a conditional equation, or an inconsistent equation.
$10x+3=8x+3$

Objective #5: Solve applied problems using formulas.

✔ *Solved Problem #5*

5. It has been shown that persons with a low sense of humor have higher levels of depression in response to negative life events than those with a high sense of humor. This can be modeled by the following formulas:

Low-Humor Group: $D = \frac{10}{9}x + \frac{53}{9}$

High-Humor Group: $D = \frac{1}{9}x + \frac{26}{9}$

where *x* represents the intensity of a negative life event (from a low of 1 to a high of 10) and *D* is the level of depression in response to that event.

If the low-humor group averages a level of depression of 10 in response to a negative life event, what is the intensity of that event?

Low-Humor Group: $D = \frac{10}{9}x + \frac{53}{9}$

$$10 = \frac{10}{9}x + \frac{53}{9}$$
$$9 \cdot 10 = 9\left(\frac{10}{9}x + \frac{53}{9}\right)$$
$$90 = 10x + 53$$
$$90 - 53 = 10x + 53 - 53$$
$$37 = 10x$$
$$\frac{37}{10} = \frac{10x}{10}$$
$$3.7 = x$$
$$x = 3.7$$

The formula indicates that if the low-humor group averages a level of depression of 10 in response to a negative life event, the intensity of that event is 3.7.

Pencil Problem #5

5. The formula $C = 1.9x + 125.5$ can be used to model the cost, *C*, *x* years after 2010 of what cost \$100 in 1999. Use the model to determine in which year the cost will be \$160 for what cost \$100 in 1999.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. {11} *(1.2 #4)* **1b.** {6} *(1.2 #23)* **2.** $\left\{\frac{33}{2}\right\}$ *(1.2 #35)*

3a. $\left\{\frac{1}{2}\right\}$ *(1.2 #41)* **3b.** $\varnothing$ *(1.2 #51)*

4a. The solution set is $\{x \mid x \text{ is a real number}\}$ or $(-\infty, \infty)$ or $\mathbb{R}$. The equation is an identity. *(1.2 #71)*

4b. The solution set is $\{0\}$. The equation is conditional. *(1.2 #75)*

5. 2028 *(1.2 #111)*

Section 1.3
Models and Applications

Counting Your Money!

From how much you can expect to earn with a bachelor's degree to how much you need to save each month for retirement, mathematical models can help you plan your finances.
In this section, you will see applications that involve salaries based on level of education, investing money in two or more accounts to obtain a specified return each year, and the cost of a health club membership.

Objective #1: Use linear equations to solve problems.

✔ *Solved Problem #1*

1a. The median yearly before-tax income of a full-time worker with a bachelor's degree exceeds that of a full-time worker with an associate's degree by $15 thousand. The median yearly before-tax income of a full-time worker with a master's degree exceeds that of a full-time worker with an associate's degree by $30 thousand. Combined, three full-time workers with each of these educational attainments earn $195 thousand before taxes. Find the median yearly before-tax income of full-time workers with each of these levels of education.

Since the salaries for a full-time worker with bachelor's and master's degrees are compared to salaries of a full-time worker with associate's degrees, we let x = the average salary of a full-time worker with an associate's degree.

$x + 15$ = the average salary of a full-time worker with a bachelor's degree
$x + 30$ = the average salary of a full-time worker with a master's degree

Since the combined salary is $195 thousand, we add the three salaries and set the sum equal to 195 and solve for x.

$$\begin{aligned} x+(x+15)+(x+30) &= 195 \\ x+x+15+x+30 &= 195 \\ 3x+45 &= 195 \\ 3x &= 150 \\ x &= 50 \end{aligned}$$

$x + 15 = 50 + 15 = 65$
$x + 30 = 50 + 30 = 80$

The average salaries are $50 thousand for an associate's degree, $65 thousand for a bachelor's degree, and $80 thousand for a master's degree.
You should verify that these salaries are $195 thousand combined.

Pencil Problem #1

1a. According to the American Bureau of Labor Statistics, you will devote 37 years to sleeping and watching TV. The number of years sleeping will exceed the number of years watching TV by 19. Over your lifetime, how many years will you spend on each of these activities?

1b. You drive up to a toll plaza and find booths with attendants where you can pay the toll by cash or credit card. With this option, the toll is \$5 each time you cross the bridge. The attendant gives you the option of buying a bar-coded decal for \$25; with the decal, you get 25% off the normal toll of \$5 for each crossing. Find the number of times you would need to cross the bridge for the costs of the two options to be the same.

Let x = the number of bridge crossings for which the costs will be the same.

The cost without the decal is \$5 times the number of crossings, x:

$5x$

The cost with the decal is \$25 plus $(0.75 \cdot \$5)$, or\$3.75 times the number of crossings, x:

$25 + 3.75x$

Since we are interested in the costs being the same, we set the costs equal and solve for x.

$$\begin{aligned} 5x &= 25 + 3.75x \\ 1.25x &= 25 \\ x &= 20 \end{aligned}$$

The costs are the same for 20 bridge crossings.
You should verify that the costs are the same for 20 bridge crossings.

1b. You are choosing between two gyms. Gym A offers membership for a fee of \$40 plus a monthly fee of \$25. Gym B offers membership for a fee of \$15 plus a monthly fee of \$30. After how many months will the total cost at each gym be the same?

1c. You inherited $50,000 with the stipulation that for the first year the money had to be invested in two funds paying 0.9% and 1.1% annual interest. How much did you invest at each rate if the total interest earned for the year was $515?

Let x = the amount invested at 0.9%.

Since a total of $50,000 is invested, 50,000 – x is invested at 1.1%.

Note that $x + (50{,}000 - x) = 50{,}000$.

The interest on the amount invested at 0.9% is $0.009x$, using $I = Pr$. The interest on the amount invested at 1.1% is $0.011(50{,}000 - x)$. The total interest is 515.

$$
\begin{aligned}
0.009x + 0.011(50{,}000 - x) &= 515 \\
0.009x + 550 - 0.011x &= 515 \\
-0.02x + 550 &= 515 \\
-0.002x &= -35 \\
x &= \frac{-35}{-0.002} \\
x &= 17{,}500 \\
50{,}000 - x &= 32{,}500
\end{aligned}
$$

$17,500 was invested at 0.9% and $32,500 was invested at 1.1%.

You should verify that the resulting interest is $515.

1c. You invested $20,000 in two accounts paying 1.45% and 1.59% annual interest. If the total interest earned for the year was $307.50, how much was invested at each rate?

Objective #2: Solve a formula for a variable.

✔ *Solved Problem #2*

2. Solve each formula for the specified variable.

2a. $P = 2l + 2w$ for w

$$P = 2l + 2w$$
$$P - 2l = 2l - 2l + 2w$$
$$P - 2l = 2w$$
$$\frac{P - 2l}{2} = \frac{2w}{2}$$
$$\frac{P - 2l}{2} = w \text{ or } w = \frac{P - 2l}{2}$$

2b. $P = C + MC$ for C

Begin by factoring out C on the right.

$$P = C + MC$$
$$P = C(1 + M)$$
$$\frac{P}{1 + M} = \frac{C(1 + M)}{1 + M}$$
$$\frac{P}{1 + M} = C \text{ or } C = \frac{P}{1 + M}$$

Pencil Problem #2

2. Solve each formula for the specified variable.

2a. $T = D + pm$ for p

2b. $IR + Ir = E$ for I

<u>**Answers** for Pencil Problems *(Textbook Exercise references in parentheses)*</u>:

1a. TV: 9 years; sleeping: 28 years *(1.3 #1)*

1b. 5 months *(1.3 #9)*

1c. \$7500 was invested at 1.45% and \$12,500 was invested at 1.59% *(1.3 #21)*

2a. $p = \frac{T - D}{m}$ *(1.3 #43)* **2b.** $I = \frac{E}{R + r}$ *(1.3 #51)*

Section 1.4
Complex Numbers

Why Study Something if it is *IMAGINARY*???

Great Question!
The numbers that we study in this section were given the name "*imaginary*" at a time when mathematicians believed such numbers to be useless.

Since that time, many *real-life* applications for so-called imaginary numbers have been discovered, but the name they were originally given has endured.

Objective #1: Add and subtract complex numbers.

✔ ***Solved Problem #1***	***Pencil Problem #1***
1a. Add: $(5-2i)+(3+3i)$ $(5-2i)+(3+3i)=5-2i+3+3i$ $=8+i$	**1a.** Add: $(7+2i)+(1-4i)$
1b. Subtract: $(2+6i)-(12-i)$ $(2+6i)-(12-i)=2+6i-12+i$ $=-10+7i$	**1b.** Subtract: $(3+2i)-(5-7i)$

Objective #2: Multiply complex numbers.

✔ *Solved Problem #2*

2a. Multiply: $(5+4i)(6-7i)$

$$\begin{aligned}(5+4i)(6-7i) &= 30-35i+24i-28i^2\\ &= 30-35i+24i-28(-1)\\ &= 30+28-35i+24i\\ &= 58-11i\end{aligned}$$

2b. Multiply: $7i(2-9i)$

$$\begin{aligned}7i(2-9i) &= 7i\cdot 2-7i\cdot 9i\\ &= 14i-63i^2\\ &= 14i-63(-1)\\ &= 63+14i\end{aligned}$$

Pencil Problem #2

2a. Multiply: $(-5+4i)(3+i)$

2b. Multiply: $-3i(7i-5)$

Objective #3: Divide complex numbers.

✔ *Solved Problem #3*

3. Divide and express the result in standard form: $\dfrac{5+4i}{4-i}$

Multiply the numerator and the denominator by the conjugate of the denominator, 4 + *i*.

$$\begin{aligned}\frac{5+4i}{4-i} &= \frac{(5+4i)}{(4-i)}\cdot\frac{(4+i)}{(4+i)}\\ &= \frac{20+5i+16i+4i^2}{16+1}\\ &= \frac{20+21i+4(-1)}{16+1}\\ &= \frac{16+21i}{17}\\ &= \frac{16}{17}+\frac{21}{17}i\end{aligned}$$

Pencil Problem #3

3. Divide and express the result in standard form: $\dfrac{2+3i}{2+i}$

Objective #4: Simplifying square roots of negative numbers.

✔ *Solved Problem #4*

4. Simplify and write the result in standard form.

4a. $\sqrt{-121}$

$$\sqrt{-121} = i\sqrt{121} = 11i$$

4b. $\sqrt{-80}$

$$\sqrt{-80} = i\sqrt{80} = 4i\sqrt{5}$$

4c. $\sqrt{7^2 - 4\cdot 5\cdot 4}$

$$\sqrt{7^2 - 4\cdot 5\cdot 4} = \sqrt{-31} = i\sqrt{31}$$

Pencil Problem #4

4. Simplify and write the result in standard form.

4a. $\sqrt{-49}$

4b. $\sqrt{-108}$

4c. $\sqrt{3^2 - 4\cdot 2\cdot 5}$

Objective #5: Perform operations with square roots of negative numbers.

✔ Solved Problem #5

5. Perform the indicated operations and write the result in standard form.

5a. $\sqrt{-27}+\sqrt{-48}$

$$\begin{aligned}\sqrt{-27}+\sqrt{-48} &= i\sqrt{27}+i\sqrt{48}\\ &= i\sqrt{9\cdot 3}+i\sqrt{16\cdot 3}\\ &= 3i\sqrt{3}+4i\sqrt{3}\\ &= 7i\sqrt{3}\end{aligned}$$

5b. $(-2+\sqrt{-3})^2$

$$\begin{aligned}(-2+\sqrt{-3})^2 &= (-2+i\sqrt{3})^2\\ &= (-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2\\ &= 4-4i\sqrt{3}+3i^2\\ &= 4-4i\sqrt{3}+3(-1)\\ &= 1-4i\sqrt{3}\end{aligned}$$

5c. $\dfrac{-14+\sqrt{-12}}{2}$

$$\begin{aligned}\frac{-14+\sqrt{-12}}{2} &= \frac{-14+i\sqrt{12}}{2}\\ &= \frac{-14+2i\sqrt{3}}{2}\\ &= \frac{-14}{2}+\frac{2i\sqrt{3}}{2}\\ &= -7+i\sqrt{3}\end{aligned}$$

Pencil Problem #5

5. Perform the indicated operations and write the result in standard form.

5a. $\sqrt{-64}-\sqrt{-25}$

5b. $(-3-\sqrt{-7})^2$

5c. $\dfrac{-8+\sqrt{-32}}{24}$

Objective #6: Simplify powers of *i*.

✔ *Solved Problem #6*

6. Simplify.

6a. i^{65}

Dividing the exponent 65 by 4, the quotient is 16 and the remainder is 1, so $65 = 4 \cdot 16 + 1$.

$$\begin{aligned} i^{65} &= i^{4\cdot 16+1} \\ &= \left(i^4\right)^{16} i^1 \\ &= (1)^{16}\, i \\ &= i \end{aligned}$$

6b. i^{72}

Dividing the exponent 72 by 4, the quotient is 18 and the remainder is 0, so $72 = 4 \cdot 18$

$$\begin{aligned} i^{72} &= i^{4\cdot 18} \\ &= \left(i^4\right)^{18} \\ &= (1)^{18} \\ &= 1 \end{aligned}$$

Pencil Problem #6

6. Simplify.

6a. i^{31}

6b. i^{114}

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $8 - 2i$ *(1.4 #1)* **1b.** $-2 + 9i$ *(1.4 #3)*

2a. $-19 + 7i$ *(1.4 #11)* **2b.** $21 + 15i$ *(1.4 #9)*

3. $\frac{7}{5}+\frac{4}{5}i$ *(1.4 #27)*

4a. $7i$ *(1.4 #29)* **4b.** $6i\sqrt{3}$ *(1.4 #31)* **4c.** $i\sqrt{31}$ *(1.4 #33)*

5a. $3i$ *(1.4 #37)* **5b.** $2+6i\sqrt{7}$ *(1.4 #43)* **5c.** $-\frac{1}{3}+i\frac{\sqrt{2}}{6}$ *(1.4 #45)*

6a. i *(1.4 #53)* **6b.** -1 *(1.4 #57)*

Section 1.5
Quadratic Equations

Maybe I Should Ride the Bus Instead

Did you know that the likelihood that a driver will be involved in a fatal crash decreases with age until about age 45 and then increases after that? Formulas that model data that first decrease and then increase contain a variable squared. When we use these models to answer questions about the data, we often need to find the solutions of a *quadratic equation*.

Unlike linear equations, quadratic equations may have exactly two distinct solutions. Thus, when we find the age at which drivers are involved in 3 fatal crashes per 100 million miles driven, we will find two different ages, one less 45 and the other greater than 45.

Objective #1: Solve quadratic equations by factoring.

✔ *Solved Problem #1*

1. Solve by factoring.

1a. $3x^2 = 9x$

$$3x^2 = 9x$$
$$3x^2 - 9x = 0$$
$$3x(x-3) = 0$$
$$3x = 0 \text{ or } x - 3 = 0$$
$$x = 0 \text{ or } \quad x = 3$$

The solution set is $\{0, 3\}$.

1b. $2x^2 = 1 - x$

$$2x^2 = 1 - x$$
$$2x^2 + x = 1$$
$$2x^2 + x - 1 = 0$$
$$(2x-1)(x+1) = 0$$
$$2x - 1 = 0 \text{ or } x + 1 = 0$$
$$2x = 1 \text{ or } \quad x = -1$$
$$x = \frac{1}{2}$$

The solution set is $\left\{-1, \frac{1}{2}\right\}$.

Pencil Problem #1

1. Solve by factoring.

1a. $3x^2 + 12x = 0$

1b. $x^2 = 8x - 15$

Objective #2: Solve quadratic equations by the square root property.

✔ *Solved Problem #2*

2. Solve by the square root property.

2a. $3x^2 - 21 = 0$

$$3x^2 - 21 = 0$$
$$3x^2 = 21$$
$$x^2 = 7$$
$$x = \pm\sqrt{7}$$

The solution set is $\left\{-\sqrt{7},\ \sqrt{7}\right\}$.

2b. $5x^2 + 45 = 0$

$$5x^2 + 45 = 0$$
$$5x^2 = -45$$
$$x^2 = -9$$
$$x = \pm\sqrt{-9}$$
$$x = \pm 3i$$

The solution set is $\{-3i, 3i\}$.

2c. $(x+5)^2 = 11$

$$(x+5)^2 = 11$$
$$x + 5 = \pm\sqrt{11}$$
$$x = -5 \pm \sqrt{11}$$

The solution set is $\{-5+\sqrt{11},\ -5-\sqrt{11}\}$.

Pencil Problem #2

2. Solve by the square root property.

2a. $5x^2 + 1 = 51$

2b. $2x^2 - 5 = -55$

2c. $3(x-4)^2 = 15$

Objective #3: Solve quadratic equations by completing the square.

✔ *Solved Problem #3*

3a. What term should be added to the binomial $x^2 + 6x$ so that it becomes a perfect square trinomial? Write and factor the trinomial.

The coefficient of the x-term of $x^2 + 6x$ is 6.

Half of 6 is 3, and 3^2 is 9, which should be added to the binomial.

The result is a perfect square trinomial.

$$x^2 + 6x + 9 = (x+3)^2$$

Pencil Problem #3

3a. What term should be added to the binomial $x^2 - 10x$ so that it becomes a perfect square trinomial? Write and factor the trinomial.

3b. Solve by completing the square: $x^2+4x-1=0$

$$x^2+4x-1=0$$
$$x^2+4x\quad=1$$

Half of 4 is 2, and 2^2 is 4, which should be added to both sides.

$$x^2+4x+4=1+4$$
$$x^2+4x+4=5$$
$$(x+2)^2=5$$
$$x+2=\sqrt{5} \quad \text{or} \quad x+2=-\sqrt{5}$$
$$x=-2+\sqrt{5} \qquad x=-2-\sqrt{5}$$

The solution set is $\{-2\pm\sqrt{5}\}$.

3b. Solve by completing the square: $x^2-6x-11=0$

3c. Solve by completing the square: $2x^2+3x-4=0$

Since the coefficient of x^2 is 2, begin by dividing both sides by 2.

$$2x^2+3x-4=0$$
$$x^2+\frac{3}{2}x-2=0$$
$$x^2+\frac{3}{2}x=2$$

Half of the coefficient of x is $\frac{1}{2}\left(\frac{3}{2}\right)=\frac{3}{4}$, and

$\left(\frac{3}{4}\right)^2=\frac{9}{16}$. Add $\frac{9}{16}$ to both sides.

$$x^2+\frac{3}{2}x+\frac{9}{16}=2+\frac{9}{16}$$
$$\left(x+\frac{3}{4}\right)^2=\frac{41}{16}$$
$$x+\frac{3}{4}=\pm\sqrt{\frac{41}{16}}$$
$$x=-\frac{3}{4}\pm\frac{\sqrt{41}}{4}$$
$$x=\frac{-3\pm\sqrt{41}}{4}$$

The solution set is $\left\{\frac{-3+\sqrt{41}}{4}, \ \frac{-3-\sqrt{41}}{4}\right\}$.

3c. Solve by completing the square: $3x^2-2x-2=0$

Objective #4: Solve quadratic equations using the quadratic formula.

✔ ***Solved Problem #4***

4a. Solve using the quadratic formula: $2x^2+2x-1=0$

The equation is in the form $ax^2+bx+c=0$, where $a = 2$, $b = 2$, and $c = -1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-2\pm\sqrt{2^2-4(2)(-1)}}{2(2)}$$
$$=\frac{-2\pm\sqrt{4+8}}{4}$$
$$=\frac{-2\pm\sqrt{12}}{4}$$
$$=\frac{-2\pm2\sqrt{3}}{4}$$
$$=\frac{2(-1\pm\sqrt{3})}{4}$$
$$=\frac{-1\pm\sqrt{3}}{2}$$

The solution set is $\left\{\frac{-1+\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}\right\}$.

4b. Solve using the quadratic formula: $x^2-2x+2=0$

The equation is in the form $ax^2+bx+c=0$, where $a = 1$, $b = -2$, and $c = 2$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(2)}}{2(1)}$$
$$=\frac{2\pm\sqrt{4-8}}{2}$$
$$=\frac{2\pm\sqrt{-4}}{2}$$
$$=\frac{2\pm2i}{2}$$
$$=\frac{2(1\pm i)}{2}$$
$$=1\pm i$$

The solution set is $\{1 + i, 1 - i\}$.

Pencil Problem #4

4a. Solve using the quadratic formula: $3x^2-3x-4=0$

4b. Solve using the quadratic formula: $x^2-6x+10=0$

Objective #5: Use the discriminant to determine the number and type of solutions.

✔ Solved Problem #5

5a. Compute the discriminant and determine the number and type of solutions: $x^2+6x+9=0$

$$b^2-4ac = 6^2-4(1)(9)$$
$$= 0$$

Since the discriminant is zero, there is one (repeated) real rational solution.

5b. Compute the discriminant and determine the number and type of solutions: $2x^2-7x-4=0$

$$b^2-4ac = (-7)^2-4(2)(-4)$$
$$= 81$$

Since the discriminant is positive and a perfect square, there are two real rational solutions.

5c. Compute the discriminant and determine the number and type of solutions: $3x^2-2x+4=0$

$$b^2-4ac = (-2)^2-4(3)(4)$$
$$= -44$$

Since the discriminant is negative, there is no real solution. There are imaginary solutions that are complex conjugates.

Pencil Problem #5

5a. Compute the discriminant and determine the number and type of solutions: $x^2-2x+1=0$

5b. Compute the discriminant and determine the number and type of solutions: $x^2-4x-5=0$

5c. Compute the discriminant and determine the number and type of solutions: $4x^2-2x+3=0$

Objective #6: Determine the most efficient method to use when solving a quadratic equation.

✔ Solved Problem #6

6. What is the most efficient method for solving a quadratic equation of the form $ax^2+c=0$?

The most efficient method is to solve for x^2 and apply the square root property.

Pencil Problem #6

6. What is the most efficient method for solving a quadratic equation of the form $u^2=d$, where u is a first-degree polynomial?

Objective #7: Solve problems modeled by quadratic equations.

✔ *Solved Problem #7*

7. The function $P(A) = 0.01A^2 + 0.05A + 107$ models a woman's normal systolic blood pressure, $P(A)$, at age A. Use this function to find the age, to the nearest year, of a woman whose normal systolic blood pressure is 115 mm Hg.

$$P(A) = 0.01A^2 + 0.05A + 107$$
$$115 = 0.01A^2 + 0.05A + 107$$
$$0 = 0.01A^2 + 0.05A - 8$$

$a = 0.01 \quad b = 0.05 \quad c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-0.05 \pm \sqrt{0.05^2 - 4(0.01)(-8)}}{2(0.01)}$$
$$= \frac{-0.05 \pm \sqrt{0.3225}}{0.02}$$
$$x = \frac{-0.05 + \sqrt{0.3225}}{0.02} \quad \text{or} \quad x = \frac{-0.05 - \sqrt{0.3225}}{0.02}$$
$x \approx 26$ ~~$x \approx -31$~~

A woman's normal systolic blood pressure is 115 mm at about 26 years of age.

Pencil Problem #7

7. The number of fatal crashes per 100 million miles driven, F, for drivers of age x can be modeled by the formula $F = 0.013x^2 - 1.19x + 28.24$. What age groups are expected to be in 3 fatal crashes per 100 million miles driven.

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $\{-4, 0\}$ *(1.5 #9)* **1b.** $\{3, 5\}$ *(1.5 #3)*

2a. $\{-\sqrt{10}, \sqrt{10}\}$ *(1.5 #17)* **2b.** $\{-5i, 5i\}$ *(1.5 #19)* **2c.** $\{4+\sqrt{5}, 4-\sqrt{5}\}$ *(1.5 #23)*

3a. 25; $x^2 - 10x + 25 = (x-5)^2$ *(1.5 #37)* **3b.** $\{3+2\sqrt{5}, 3-2\sqrt{5}\}$ *(1.5 #51)*

3c. $\left\{\frac{1+\sqrt{7}}{3}, \frac{1-\sqrt{7}}{3}\right\}$ *(1.5 #63)*

4a. $\left\{\frac{3+\sqrt{57}}{6}, \frac{3-\sqrt{57}}{6}\right\}$ *(1.5 #69)* **4b.** $\{3+i, 3-i\}$ *(1.5 #73)*

5a. 0; one (repeated) real rational solution *(1.5 #79)* **5b.** 36; two real rational solutions *(1.5 #75)*

5c. −44; two imaginary solutions that are complex conjugates *(1.5 #76)*

6. The square root property *(1.5 #15-34)*

7. 33-year-olds and 58-year-olds *(1.5 #135)*

Section 1.6
Other Types of Equations

Slam Dunk!

A basketball player's hang time is the time spent in the air when shooting a basket.

In this section, we will be given a formula that involves radicals which models seconds of hang time in terms of the vertical distance of a player's jump.

Objective #1: Solve polynomial equations by factoring.

✔ *Solved Problem #1*

1a. Solve by factoring: $4x^4 = 12x^2$

$$4x^4 = 12x^2$$
$$4x^4 - 12x^2 = 0$$
$$4x^2(x^2 - 3) = 0$$
$$4x^2 = 0 \text{ or } x^2 - 3 = 0$$
$$x^2 = 0 \qquad x^2 = 3$$
$$x = 0 \qquad x = \pm\sqrt{3}$$

The solution set is $\{-\sqrt{3},\ 0,\ \sqrt{3}\}$.

1b. Solve by factoring: $2x^3 + 3x^2 = 8x + 12$

$$2x^3 + 3x^2 = 8x + 12$$
$$2x^3 + 3x^2 - 8x - 12 = 0$$
$$x^2(2x+3) - 4(2x+3) = 0$$
$$(2x+3)(x^2 - 4) = 0$$
$$2x + 3 = 0 \quad \text{or} \quad x^2 - 4 = 0$$
$$2x = -3 \qquad x^2 = 4$$
$$x = -\frac{3}{2} \qquad x = \pm 2$$

The solution set is $\left\{-2,\ -\frac{3}{2},\ 2\right\}$.

Pencil Problem #1

1a. Solve by factoring: $3x^4 - 48x^2 = 0$

1b. Solve by factoring: $3x^3 + 2x^2 = 12x + 8$

Objective #2: Solve radical equations.

✔ *Solved Problem #2*

2a. Solve: $\sqrt{x+3}+3=x$

$$\begin{aligned}\sqrt{x+3}+3&=x\\ \sqrt{x+3}&=x-3\\ (\sqrt{x+3})^2&=(x-3)^2\\ x+3&=x^2-6x+9\\ 0&=x^2-7x+6\\ 0&=(x-6)(x-1)\end{aligned}$$

$x-6=0$ or $x-1=0$

$x=6$ $\quad$ $x=1$

Check 6: $\sqrt{x+3}+3=x$

$\sqrt{6+3}+3=6$

$6=6$

Check 1: $\sqrt{x+3}+3=x$

$\sqrt{1+3}+3=1$

$5=1$

The solution set is $\{6\}$.

2b. Solve: $\sqrt{x+5}-\sqrt{x-3}=2$

$$\begin{aligned}\sqrt{x+5}-\sqrt{x-3}&=2\\ \sqrt{x+5}&=\sqrt{x-3}+2\\ \left(\sqrt{x+5}\right)^2&=\left(\sqrt{x-3}+2\right)^2\\ x+5&=x-3+4\sqrt{x-3}+4\\ x+5&=x+1+4\sqrt{x-3}\\ 4&=4\sqrt{x-3}\\ 1&=\sqrt{x-3}\\ 1^2&=\left(\sqrt{x-3}\right)^2\\ 1&=x-3\\ 4&=x\end{aligned}$$

Check:

$\sqrt{x+5}-\sqrt{x-3}=2$

$\sqrt{4+5}-\sqrt{4-3}=2$

$2=2$

The solution set is $\{4\}$.

Pencil Problem #2

2a. Solve: $\sqrt{2x+13}=x+7$

2b. Solve: $\sqrt{x-5}-\sqrt{x-8}=3$

Objective #3: Solve equations with rational exponents.

✔ *Solved Problem #3*

3a. Solve: $5x^{\frac{3}{2}}-25=0$

$$\begin{aligned}5x^{\frac{3}{2}}-25&=0\\5x^{\frac{3}{2}}&=25\\x^{\frac{3}{2}}&=5\\(x^{\frac{3}{2}})^{\frac{2}{3}}&=(5)^{\frac{2}{3}}\\x&=5^{\frac{2}{3}}\text{ or }\sqrt[3]{25}\end{aligned}$$

Check:
$$\begin{aligned}5x^{\frac{3}{2}}-25&=0\\5(5^{\frac{2}{3}})^{\frac{3}{2}}-25&=0\\5(5)-25&=0\\0&=0\end{aligned}$$

The solution set is $\{5^{\frac{2}{3}}\}$ or $\{\sqrt[3]{25}\}$.

3b. Solve: $x^{\frac{2}{3}}-8=-4$

$$\begin{aligned}x^{\frac{2}{3}}-8&=-4\\x^{\frac{2}{3}}&=4\\(x^{\frac{2}{3}})^{\frac{3}{2}}&=\pm4^{\frac{3}{2}}\\x&=\pm8\end{aligned}$$

You should verify that both –8 and 8 are solutions.
The solution set is {–8, 8}.

Pencil Problem #3

3a. Solve: $6x^{\frac{5}{2}}-12=0$

3b. Solve: $(x-4)^{\frac{2}{3}}=16$

Objective #4: Solve equations that are quadratic in form.

✔ Solved Problem #4

4a. Solve: $x^4 - 5x^2 + 6 = 0$

Let $u = x^2$.

$$x^4 - 5x^2 + 6 = 0$$
$$\left(x^2\right)^2 - 5x^2 + 6 = 0$$
$$u^2 - 5u + 6 = 0$$
$$(u-3)(u-2) = 0$$

Apply the zero product principle.

$u - 3 = 0$ or $u - 2 = 0$

$u = 3$ $\qquad u = 2$

Replace u with x^2.

$x^2 = 3$ or $x^2 = 2$

$x = \pm\sqrt{3}$ $\qquad x = \pm\sqrt{2}$

The solution set is $\left\{\pm\sqrt{2}, \pm\sqrt{3}\right\}$.

4b. Solve: $3x^{\frac{2}{3}} - 11x^{\frac{1}{3}} - 4 = 0$

Rewrite as follows.

$$3(x^{\frac{1}{3}})^2 - 11x^{\frac{1}{3}} - 4 = 0$$

Let $u = x^{\frac{1}{3}}$.

$$3u^2 - 11u - 4 = 0$$
$$(3u+1)(u-4) = 0$$

$3u + 1 = 0$ or $u - 4 = 0$

$3u = -1$ $\qquad u = 4$

$u = -\frac{1}{3}$

$x^{\frac{1}{3}} = -\frac{1}{3}$ or $x^{\frac{1}{3}} = 4$

$(x^{\frac{1}{3}})^3 = \left(-\frac{1}{3}\right)^3$ $\qquad (x^{\frac{1}{3}})^3 = (4)^3$

$x = -\frac{1}{27}$ $\qquad x = 64$

The solution set is $\left\{-\frac{1}{27},\ 64\right\}$.

Pencil Problem #4

4a. Solve: $x^4 - 5x^2 + 4 = 0$

4b. Solve: $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$

Objective #5: Solve equations involving absolute value.

✔ *Solved Problem #5*

5. Solve: $|2x-1|=5$

Rewrite without absolute value bars.
$|u|=c$ means $u=c$ or $u=-c$.

$$2x-1=5 \quad \text{or} \quad 2x-1=-5$$
$$2x=6 \qquad\qquad 2x=-4$$
$$x=3 \qquad\qquad x=-2$$

The solution set is $\{-2,3\}$.

Pencil Problem #5

5. Solve: $|x-2|=7$

Objective #6: Solve problems modeled by equations.

✔ *Solved Problem #6*

6. The formula $H=-2.3\sqrt{I}+67.6$ models weekly television viewing time, H, in hours, by annual income, I, in thousands of dollars. What annual income corresponds to 33.1 hours per week watching TV?

Substitute 33.1 for H and solve for I.

$$33.1=-2.3\sqrt{I}+67.6$$
$$-34.5=-2.3\sqrt{I}$$
$$15=\sqrt{I}$$
$$(15)^2=(\sqrt{I})^2$$
$$225=I$$

An annual income of $225,000 corresponds to 33.1 hours per week watching TV.

Pencil Problem #6

6. The formula $t=\frac{\sqrt{d}}{2}$ models a basketball player's hang time, t, in seconds, in terms of the vertical distance, d, in feet. If the hang time is 1.16 seconds, what is the vertical distance of the jump, to the nearest tenth of a foot?

Answers for Pencil Problems *(Textbook Exercise references in parentheses)*:

1a. $\{-4, 0, 4\}$ *(1.6 #1)* **1b.** $\left\{-2, -\frac{2}{3}, 2\right\}$ *(1.6 #3)*

2a. $\{-6\}$ *(1.6 #15)* **2b.** $\varnothing$ *(1.6 #25)*

3a. $\{\sqrt[5]{4}\}$ *(1.6 #35)* **3b.** $\{-60, 68\}$ *(1.6 #37)*

4a. $\{-2, -1, 1, 2\}$ *(1.6 #41)* **4b.** $\{-8, 27\}$ *(1.6 #49)*

5. $\{-5, 9\}$ *(1.6 #63)*

6. 5.4 ft *(1.6 #113)*

Section 1.7
Linear Inequalities and Absolute Values Inequalities

Are You in LOVE?

As the years go by in a relationship, three key components of love…

passion

commitment

intimacy

…progress differently over time.

Passion peaks early in a relationship and then declines.
By contrast, intimacy and commitment build gradually.

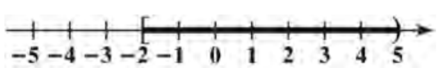
In the applications of this section of the textbook, we will use mathematics to explore the relationships among these three variables of love.

Objective #1: Use interval notation.

✔ *Solved Problem #1*

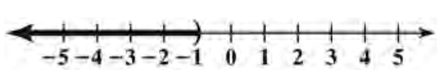
1a. Express $[-2, 5)$ in set-builder notation and graph.

$\{x \mid -2 \le x < 5\}$



1b. Express $(-\infty, -1)$ in set-builder notation and graph.

$\{x \mid x < -1\}$



Pencil Problem #1

1a. Express $(1, 6]$ in set-builder notation and graph.

1b. Express $[-3, \infty)$ in set-builder notation and graph.

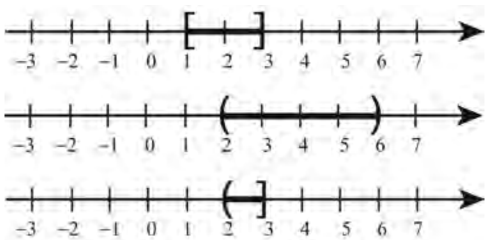
Objective #2: Find intersections and unions of intervals.

✔ *Solved Problem #2*

2. Use graphs to find each set:

2a. $[1, 3] \cap (2, 6)$

Graph each interval. The intersection consists of the portion of the number line that the two graphs have in common.



$[1, 3] \cap (2, 6) = (2, 3]$

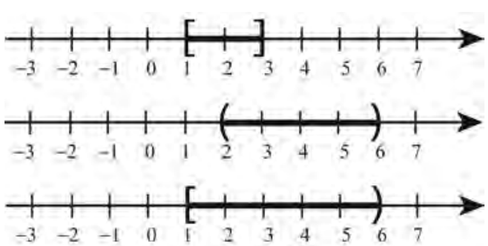
Pencil Problem #2

2. Use graphs to find each set:

2a. $(-3, 0) \cap [-1, 2]$

2b. $[1, 3] \cup (2, 6)$

Graph each interval. The union consists of the portion of the number line in either one of the intervals or the other or both.



$[1, 3] \cup (2, 6) = [1, 6)$

2b. $(-3, 0) \cup [-1, 2]$

Objective #3: Solve linear inequalities.

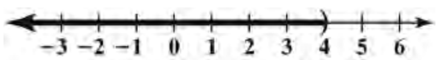
✔ *Solved Problem #3*

3a. Solve and graph the solution set on a number line:

$$3x+1>7x-15$$

$$3x+1>7x-15$$
$$-4x>-16$$
$$\frac{-4x}{-4}<\frac{-16}{-4}$$
$$x<4$$

The solution set is $(-\infty,4)$.

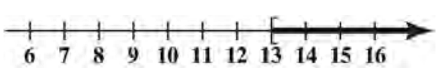


3b. Solve and graph the solution set on a number line:

$$\frac{x-4}{2}\ge\frac{x-2}{3}+\frac{5}{6}$$

$$\frac{x-4}{2}\ge\frac{x-2}{3}+\frac{5}{6}$$
$$6\left(\frac{x-4}{2}\right)\ge 6\left(\frac{x-2}{3}+\frac{5}{6}\right)$$
$$3(x-4)\ge 2(x-2)+5$$
$$3x-12\ge 2x-4+5$$
$$3x-12\ge 2x+1$$
$$x\ge 13$$

The solution set is $[13,\infty)$.



Pencil Problem #2

3a. Solve and graph the solution set on a number line:

$$-9x\ge 36$$

3b. Solve and graph the solution set on a number line:

$$\frac{x}{4}-\frac{3}{2}=\frac{x}{2}+1$$

3c. You drive up to a toll plaza and find booths with attendants, and you can pay the toll by cash or credit card. With this option, the toll is $5 each time you cross the bridge. The attendant gives you the option of buying a bar-coded decal for $25; with the decal, you get 25% off the normal toll of $5 for each crossing. Find the number of times you would need to cross the bridge to make the decal option the better deal.

Let x = the number of bridge crossings.
cost without decal = $5x$
cost with decal = $25 + 0.75(5)x$

$$5x > 25 + 3.75x$$
$$1.25x > 25$$
$$x > 20$$

Crossing more than 20 times will make the decal option the better deal.

3c. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

Objective #4: Recognize inequalities with no solution or all real numbers as solutions.

✔ ***Solved Problem #4***

4a. Solve the inequality: $3(x+1) > 3x + 2$

$$3(x+1) > 3x + 2$$
$$3x + 3 > 3x + 2$$
$$3 > 2$$

This expression is always true.

The solution set is $\mathbb{R}$ or $(-\infty, \infty)$.

4b. Solve the inequality: $x + 1 \leq x - 1$

$$x + 1 \leq x - 1$$
$$1 \leq -1$$

This expression is always false.

The solution set is $\varnothing$.

Pencil Problem #4

4a. Solve the inequality: $4(3x-2) - 3x < 3(1+3x) - 7$

4b. Solve the inequality: $5(x-2) - 3(x+4) \geq 2x - 20$

Objective #5: Solve compound inequalities.

✔ *Solved Problem #5*

5. Solve the compound inequality: $1 \le 2x+3 < 11$

$$1 \le 2x+3 < 11$$
$$1-3 \le 2x+3-3 < 11-3$$
$$-2 \le 2x < 8$$
$$\frac{-2}{2} \le \frac{2x}{2} < \frac{8}{2}$$
$$-1 \le x < 4$$

The solution set is $[-1, 4)$.

Pencil Problem #5

5. Solve the compound inequality: $-11 < 2x-1 \le -5$

Objective #6: Solve absolute value inequalities.

✔ *Solved Problem #2*

6a. Solve the inequality:

$$|x-2| < 5$$

Rewrite without absolute value bars.
$|u| < c$ means $-c < u < c$.

$$-5 < x-2 < 5$$
$$-5+2 < x-2+2 < 5+2$$
$$-3 < x < 7$$

The solution set is $(-3, 7)$.

6b. Solve the inequality:

$$-3|5x-2|+20 \ge -19$$

First, isolate the absolute value expression on one side of the inequality.

$$-3|5x-2|+20 \ge -19$$
$$-3|5x-2| \ge -39$$
$$\frac{-3|5x-2|}{-3} \le \frac{-39}{-3}$$
$$|5x-2| \le 13$$

Pencil Problem #2

6a. Solve the inequality:

$$|2x-6| < 8$$

6b. Solve the inequality:

$$|2(x-1)+4| \le 8$$

Rewrite $|5x-2| \le 13$ without absolute value bars.

$|u| \le c$ means $-c \le u \le c$.

$$-13 \le 5x - 2 \le 13$$
$$-13 + 2 \le 5x - 2 + 2 \le 13 + 2$$
$$-11 \le 5x \le 15$$
$$\frac{-11}{5} \le \frac{5x}{5} \le \frac{15}{5}$$
$$-\frac{11}{5} \le x \le 3$$

The solution set is $\left[\frac{-11}{5}, 3\right]$.

6c. Solve the inequality: $18 < |6-3x|$

Rewrite with the absolute value expression on the left.

$|6-3x| > 18$

This means the same as $6-3x < -18$ or $6-3x > 18$.

$$6-3x < -18 \quad \text{or} \quad 6-3x > 18$$
$$-3x < -24 \qquad\qquad -3x > 12$$
$$\frac{-3x}{-3} > \frac{-24}{-3} \qquad \frac{-3x}{-3} < \frac{12}{-3}$$
$$x > 8 \qquad\qquad x < -4$$

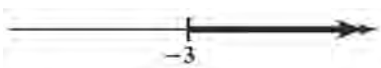
The solution set is $\{x \mid x < -4 \text{ or } x > 8\}$ or $(-\infty, -4) \cup (8, \infty)$.

6c. Solve the inequality: $1 < |2-3x|$

<u>**Answers**</u> <u>for Pencil Problems</u> <u>*(Textbook Exercise references in parentheses)*</u>:

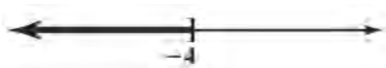
1a. $\{x \mid 1 < x \le 6\}$; 

(1.7 #1)

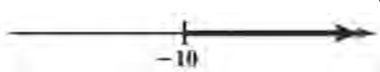
1b. $\{x \mid x \ge -3\}$; 

(1.7 #9)

2a. $[-1, 0)$ (1.7 #15) **2b.** $(-3, 2]$ *(1.7 #17)*

3a. $(-\infty, -4]$; 

(1.7 #31)

3b. $[-10, \infty)$; 

(1.7 #41) **3c.** at most 29 bags (*1.7 #129)*

4a. $(-\infty, \infty)$ *(1.7 #47)* **4b.** $\varnothing$ *(1.7 #49)*

5. $(-5, -2]$ *(1.7 #55)*

6a. $(-1, 7)$ *(1.7 #63)* **6b.** $[-5, 3]$ *(1.7 #65)* **6c.** $\left(-\infty, \frac{1}{3}\right) \cup (1, \infty)$ *(1.7 #89)*