

Formulas and Examples

Continuous Exponential Growth and Decay Model:

$$\boxed{A = A_0 e^{kt}}$$

A_0 is the initial amount of substance, A is the amount of substance after time t has passed.

k a constant that depends on the rate of growth (positive) or decay (negative).

Compound Interest Formula:

$$\boxed{A = A_0 \left(1 + \frac{r}{n}\right)^{nt}}$$

A_0 is the initial amount invested, A is the account value after time t has passed.

r is the annual interest rate. n is the number of compounding periods per year.

$$\left(1 + \frac{r}{n}\right)^{nt} \text{ let } n = xr \rightarrow \left(1 + \frac{r}{n}\right)^{nt} = \left(1 + \frac{r}{xr}\right)^{xrt} = \left[\left(1 + \frac{1}{x}\right)^x\right]^{rt} \text{ As } x \rightarrow \infty, \left(1 + \frac{1}{x}\right)^x \rightarrow e \text{ Euler's Number } \approx 2.71828 \dots$$

$$\therefore \text{ As } x \rightarrow \infty, \boxed{A = A_0 e^{rt}} \text{ Continuous Growth or Decay}$$

If R is the intensity of an earthquake (Richter Scale). A is the amplitude (measured in micrometers), and P is the period (the time of one oscillation of Earth's surface, measured in seconds), then

$$\boxed{R = \log \frac{A}{P}}$$

The more acidic a solution, the greater the concentration of hydrogen ions (moles per liter). This concentration is indicated indirectly by the pH scale, or hydrogen ion index.

If $[H^+]$ is the hydrogen ion concentration in gram-ions per liter, then $\boxed{\text{pH} = -\log [H^+]}$

Loudness of sound is measured in decibels and is calculated by a formula using the sound intensity measured in watts per square meter. The threshold intensity of sound, I_0 , is 10^{-12} watts / m^2 .

$$\boxed{L = 10 (\log I - \log I_0)}$$

$$\boxed{L = 10 (\log I + 12)}$$

$\langle EX 1 \rangle$ \$25,000 is deposited into an account earning 3.5% interest compounded quarterly for 18 years.

What will the value of the account be at the end of that time?

$\langle EX 2 \rangle$ \$25,000 is deposited into an account earning 3.5% interest compounded continuously for 18 years.

What will the value of the account be at the end of that time?

$\langle EX 3 \rangle$ The population of a city is 40,000 people, but changing economic conditions are causing the population to decrease by 2% each year. If this trend continues, then what will the population be in 10 years?

$\langle EX 4 \rangle$ In the example #3 situation how long would it take for the population to drop to 20,000 people?

⟨EX 5⟩ Over a time period of 20 days a 100 gram sample of Radon - 22 was found to decay to 2.6783 grams.
What is the half-life of Radon - 22? How long will it take the original sample to decay to 1 gram.

⟨EX 6⟩ Find the measure on the Richter Scale of an earthquake with an amplitude of 10,000 micrometers (1 centimeter) and a period of 0.1 second.

⟨EX 7⟩ What would the period need to be for the an earthquake of 10,000 micrometers need to be for a Richter Scale reading to be measured at 7.5?

⟨EX 8⟩ Find the hydrogen-ion concentration of seawater if its pH is 8.5. Write your answer in scientific form.

Logistics Growth Model :

"C" is the Ceiling Population
or Maximum Attainable Value

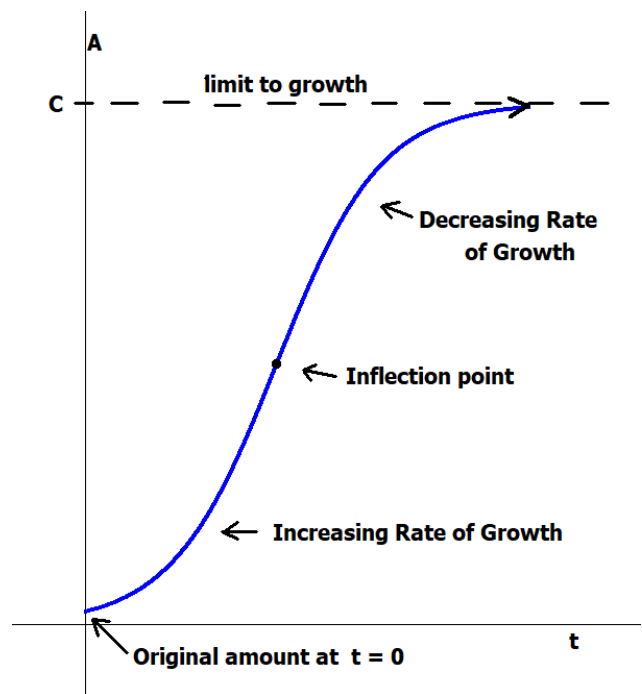
$$f(t) = \frac{C}{1 + ae^{-bt}} \quad \text{or} \quad A = \frac{C}{1 + ae^{-bt}}$$

a , b , and C are constants, with $b > 0$ and $C > 0$.

Note: As $t \rightarrow \infty$, $e^{-bt} \rightarrow 0$,
thus $f(t) \rightarrow C$ or $A \rightarrow C$

Note: When $t = 0$ you get $f(0) = \frac{C}{1 + a}$
which is the initial population or value of the function.

Note: The function has a Horizontal Asymptote $y = C$.



⟨EX 9⟩ The logistics growth function $f(t) = \frac{900}{1 + 59e^{-0.4t}}$

describes a wolf population t years after it is placed into a new area.

- How many wolves were initially introduced into the area?
- How many wolves were there in the area after 10 years after being placed in the area?
- What is the limiting size of the wolf population in the area?
- How much time will pass before the wolf population reaches 750 wolves?