MATH 125 College Algebra   Name____________________________

Rational Function: Any Function f such that

$f(x) = \dfrac{p(x)}{q(x)}$, where p and q are each polynomial functions and $q(x) \neq 0$.

For Graphing Rational Functions

Factor the Numerator and Denominator then Reduce the Function. Determine the Domain: From the **original** denominator determine the restricted values (those that cause the denominator to equal zero).

Determine any Zeros of the Function. (Domain values the cause the numerator to equal zero in the reduced form of the function.)

Determine the End Behavior. $As\ x \to \infty,\ f(x) = ?$ *and* $As\ x \to -\infty,\ f(x) = ?$

If the degree of the numerator $<$ the degree of the denominator then,

$\boxed{As\ x \to \infty,\ f(x) \to 0 \text{ and } As\ x \to -\infty,\ f(x) \to 0}$ y = 0 is a Horizontal Asymptote

If the degree of the numerator $=$ the degree of the denominator then

$Let\ \dfrac{leading\ Coef.\ of\ the\ \text{Numerator}}{leading\ Coef.\ of\ the\ \text{Denominator}} = m$ $\boxed{\begin{array}{l} As\ x \to \infty,\ f(x) \to m \\ As\ x \to -\infty,\ f(x) \to m \end{array}}$ y = m is a HA

If the degree of the numerator $>$ the degree of the denominator then

$As\ x \to \infty,\ \begin{cases} f(x) \to \infty, & \text{ratio of L.C.} > 0 \\ f(x) \to -\infty, & \text{ratio of L.C.} < 0 \end{cases}$	If the difference in the degrees is odd then $As\ x \to -\infty$, the graph will rise or fall opposite of $As\ x \to \infty$. If the difference in the degrees is even then $As\ x \to -\infty$, the graph will rise or fall similar to $As\ x \to \infty$.

Then divide the numerator and denominator to determine any Oblique (Slant) Asymptote or other behavior.

Determine the x-intercept(s) (let the function equal zero and solve of x) and determine the y-intercept (let x = 0 and determine the y value).

Determine any Veritcal Asymptotes: x = restricted values in the reduced form of the function the graph will have Vertical Asymptotes. The graph will go up without bound or down without bound on either side of a Vertical Asymptote

If $y = c$ is a Vertical Asymptote, then

$\boxed{As\ x \to c^-,\ f(x) \to \infty \ or\ -\infty \quad \text{and} \quad As\ x \to c^+,\ f(x) \to \infty \ or\ -\infty.}$

Determine any Removable Discontinuities - values of *x* where the original function is undefined (the denominator equal zero) but the reduced form is defined (denominator is not equal to zero). There will be a “hole” in the graph at these x values. Get the y coordinate for these “holes” by substituting the x value in the reduced form of the equation.

Sketch the curve

$$g(x) = \frac{1}{x}$$

$$f(x) = \frac{x}{x+3}$$

$$t(x) = \frac{2x(x-2)(x-3)}{(x-3)(x-4)^2} \Rightarrow t(x) = \frac{2x(x-2)}{(x-4)^2}$$

$$f(x) = \frac{x^2+2}{x-2}$$