MATH 125 College Algebra   Name____________________________

Division of Polynomials   Complete each division

$$\frac{18a^5c^3 + 21a^4c - 24a^3c^2}{-3a^3c}$$

$$\frac{-12m^4n^2 + 30m^2n^5 + 6m^3n^2 - m^4n^3}{-6m^3n^2}$$

$$\frac{x^3 + 8x^2 - 5x - 60}{x + 3}$$

$$\frac{6x^3 - x^2 - 23x + 28}{3x + 7}$$

Nested forms:

$$q(x) = 7x^2 - x + 10$$
$$= \left(7x^2 - x\right) + 10$$
$$= x\left(7x - 1\right) + 10$$
$$= x\left(\left(7x\right) - 1\right) + 10$$
$$q(x) = x\left(x\left(7\right) - 1\right) + 10$$

$$t(x) = 2x^3 - 25x + 15$$
$$t(x) = \left(2x^3 + 0x^2 - 25x\right) + 15$$
$$t(x) = x\left(2x^2 + 0x - 25\right) + 15$$
$$t(x) = x\left(\left[2x^2 + 0x\right] - 25\right) + 15$$
$$t(x) = x\left(x\left[2x + 0\right] - 25\right) + 15$$
$$t(x) = x\left(x\left[\left(2x\right) + 0\right] - 25\right) + 15$$
$$t(x) = x\left(x\left[x\left(2\right) + 0\right] - 25\right) + 15$$

$$p(x) = 5x^4 - 2x^3 + 6x^2 - 4x + 1$$
$$= \left(5x^4 - 2x^3 + 6x^2 - 4x\right) + 1$$
$$= x\left(5x^3 - 2x^2 + 6x - 4\right) + 1$$
$$= x\left(\left[5x^3 - 2x^2 + 6x\right] - 4\right) + 1$$
$$= x\left(x\left[5x^2 - 2x + 6\right] - 4\right) + 1$$
$$= x\left(x\left[\left(5x^2 - 2x\right) + 6\right] - 4\right) + 1$$
$$= x\left(x\left[x\left(5x - 2\right) + 6\right] - 4\right) + 1$$
$$= x\left(x\left[x\left(\left[5x\right] - 2\right) + 6\right] - 4\right) + 1$$
$$p(x) = x\left(x\left[x\left(x\left[5\right] - 2\right) + 6\right] - 4\right) + 1$$

Synthetic Substitution:

$q(2) = 7 \xrightarrow{\cdot 2} 14 \xrightarrow{-1} 13 \xrightarrow{\cdot 2} 26 \xrightarrow{+10} 36$

$\therefore\ q(2) = 36$

$p(3) = 5 \xrightarrow{\cdot 3} 15 \xrightarrow{-2} 13 \xrightarrow{\cdot 3} 39 \xrightarrow{+6} 45 \xrightarrow{\cdot 3} 135 \xrightarrow{-4} 131 \xrightarrow{\cdot 3} 393 \xrightarrow{+1} 394$

$\therefore\ p(3) = 394$

$$\begin{array}{r|rrr}
2 & 7 & -1 & 10 \\
 & & 14 & 26 \\
\hline
 & 7 & 13 & \big| \; 36 = q(2)
\end{array}$$

$$\begin{array}{r|rrrrr}
3 & 5 & -2 & 6 & -4 & 1 \\
 & & 15 & 39 & 135 & 393 \\
\hline
 & 5 & 13 & 45 & 131 & \big| \; 394 = p(3)
\end{array}$$

$t(-4) = 2 \xrightarrow{\cdot {}^{-}4} {}^{-}8 \xrightarrow{+0} {}^{-}8 \xrightarrow{\cdot {}^{-}4} 32 \xrightarrow{-25} 7 \xrightarrow{\cdot {}^{-}4} {}^{-}28 \xrightarrow{+15} {}^{-}13$

$\therefore\ t(-4) = {}^{-}13$

$$\begin{array}{r|rrrr}
-4 & 2 & 0 & -25 & 15 \\
 & & -8 & 32 & -28 \\
\hline
 & 2 & -8 & 7 & \big| \; -13 = t(-4)
\end{array}$$

FACTOR THEOREM: $f(c)=0$ iff $(x-c)$ is a factor of $f(x)$.

$$\frac{3x^4-53x^2+18x+10}{x-4}$$

$$\frac{10x^5-44x^4+18x^2-60x-6}{5x+3}$$

<u>The Remainder Theorem</u>

If a polynomial $f(x)$ is divided by $(x-c)$, then the remainder is $f(c)$.

Note: If $f(c) = 0 \Rightarrow f(x) = (x-c)\,q(x)$

$\therefore$ If $f(c) = 0$, then $(x-c)$ is a factor of $f(x)$.

$P(x) = x^4 - x^3 - 7x^2 + 6x - 9$

Determine the value of $P(2)$ $\quad \dfrac{x^4 - x^3 - 7x^2 + 6x - 9}{x-2} =$

Determine the value of $P(3)$ $\quad \dfrac{x^4 - x^3 - 7x^2 + 6x - 9}{x-3} =$