Linear Transformations:

Apply effects of C before B and A before D.

Effects the Domain (*Inversely*) . . . . "x" variable position - $\langle$ *Horizontal Change* $\rangle$

C:

if $\underline{\underline{C < 1}}$ the function is "Translated Horizontally (shift or slide) C units to the right "

if $\underline{\underline{C > 1}}$ the function is "Translated Horizontally (shift or slide) C units to the left "

if B is negative the function is "Reflected Horizontally (flipped) over the $y-$axis"

B: if $\underline{\underline{|B| > 1}}$ the function is "Compressed Horizontally (squeezed) toward the $y-$axis by a factor of $\left|\frac{1}{B}\right|$ "

if $\underline{\underline{0 < |B| < 1}}$ the function is "Expanded Horizontally (streched) from the $y-$axis by a factor of $\left|\frac{1}{B}\right|$ "

$$y = A f\left(B x + C \right) + D$$

Apply effects A before D.

Effects the Range (*Directly*) . . . . "y" variable position - $\langle$ *Vertical Change* $\rangle$

if A is negative the function is "Reflected Vertically (flipped) over the $x-$axis"

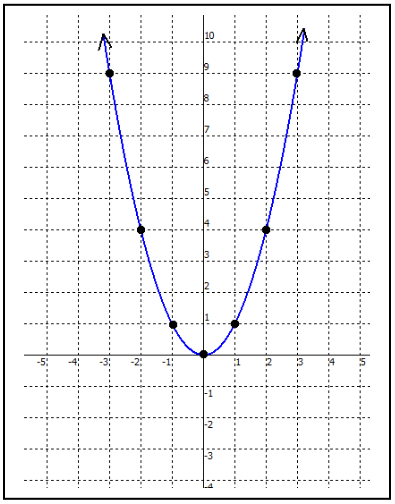
A: if $\underline{\underline{|A| > 1}}$ the function is "Expanded Vertically (stretched) from the $x-$axis by a factor of $|A|$ "

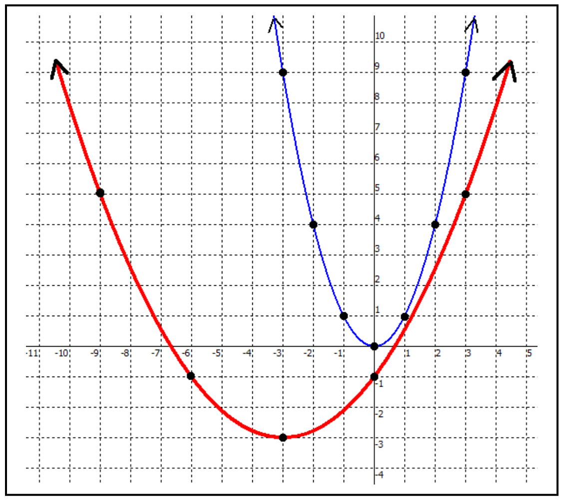
if $\underline{\underline{0 < |A| < 1}}$ the function is "Compressed Vertically (streched) toward the $y-$axis by a factor of $|A|$ "

D:

if $\underline{\underline{D < 1}}$ the function is "Translated Vertically (shift or slide) D units down "

if $\underline{\underline{D > 1}}$ the function is "Translated Vertically (shift or slide) D units up "





$$y=2\left(\frac{1}{3}x+1\right)^2-3$$

$$y=(x)^2$$

$\cdot 3 \swarrow$	$-1 \swarrow$	x	y	$\searrow \cdot 2$	$\searrow -3$
−3	−1	0	0	0	−3
−6	−2	−1	1	2	−1
0	0	1	1	2	−1
−9	−3	−2	4	8	5
3	1	2	4	8	5
−12	−4	−3	9	18	15
−6	2	3	9	18	15

1. Describe the graph of $y=2\left(\frac{1}{3}x+1\right)^2-3$ as a transformation from the parent function $R(x)=x^2$.

1st Translate (Shift or Slide) the points of the parent function 1 units to the left.

2nd Expand (Stretch) the points of the graph horizontally to positions 3times as far from the y-axis.

3rd Expand (Stretch) the points of the graph vertically from the x-axis to positions 2 times as far from the x-axis.

4th Translate (Shift or Slide) the points of the graph 3 units down.

2. Describe the graph of $y = -\frac{1}{2}P(3x-9)+1$ as a transformation from the parent function .

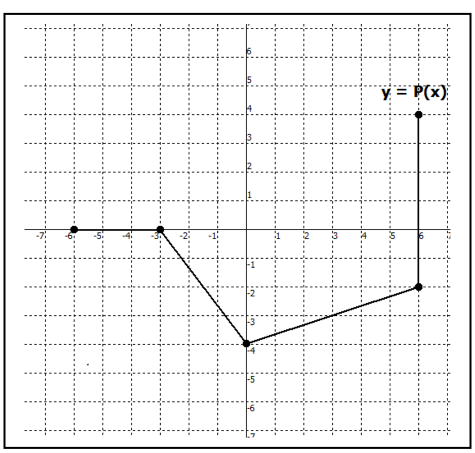
$y = P(x)$.

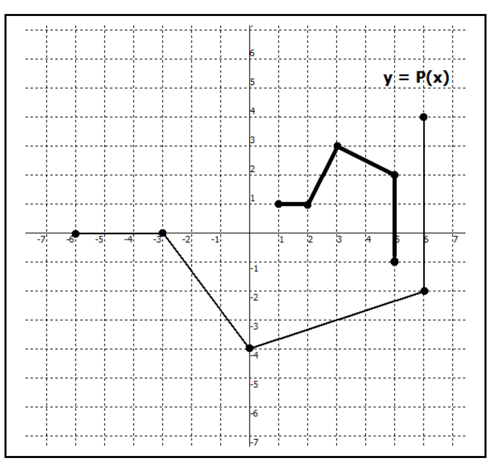
1st Translate (Shift or Slide) the points of the parent function 9 units to the right.

2nd Compress (Squeeze) the points of the graph horizontally to positions $\frac{1}{3}$ as far from the y-axis.

3rd Reflect the points of the graph over the x-axis, then Compress (Squeeze) them vertically to positions $\frac{1}{2}$ as far from the x-axis.

4th Translate (Shift or Slide) the points of the graph 1 unit up.





$$y = -\frac{1}{2}P(3x-9)+1$$

$$y=P(x)$$

$\cdot\frac{1}{3}\swarrow$	$+9\swarrow$	x	y	$\searrow \cdot -\frac{1}{2}$	$\searrow +1$
1	3	−6	0	0	1
2	6	−3	0	0	1
3	9	0	−4	2	3
5	15	6	−2	1	2
5	15	6	4	−2	−1

3. Describe the graph of $h(x) = 4\sqrt{2x - 3} - 1$ as a transformation from the parent function $R(x) = \sqrt{x}$.

1st Translate (Shift or Slide) the points of the parent function 3 units to the right.

2nd Compress (Squeeze) the points of the graph horizontally to positions $\frac{1}{2}$ as far from the y-axis.

3rd Expand (Stretch) the points of the graph vertically from the x-axis to positions 4 times as far from the x-axis.

4th Translate (Shift or Slide) the points of the graph 1 units down.

4. Describe the graph of $g(x) = -3\left|\frac{1}{2}x + 1\right| - 2$ as a transformation from the parent function $A(x) = |x|$.

1st Translate (Shift or Slide) the points of the parent function 1 units to the left.

2nd Expand (Stretch) the points of the graph horizontally to positions 2 times as far from the y-axis.

3rd Reflect the points of the graph over the x-axis, then Expand (Stretch) them vertically to positions 3 times as far from the x-axis.

4th Translate (Shift or Slide) the points of the graph 2 units down.

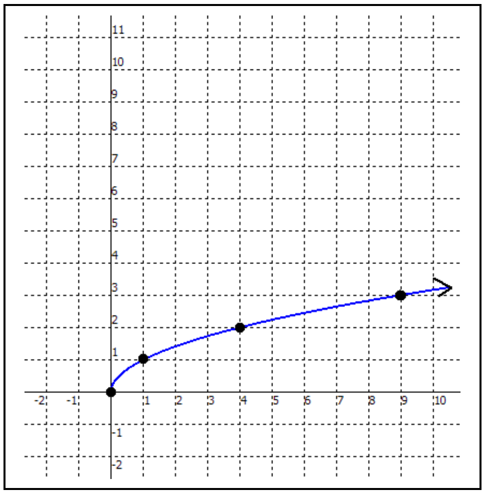
5. Describe the graph of $f(x) = \frac{1}{2}\left(-\frac{4}{3}x - 5\right)^3 + 6$ as a transformation from the parent function $C(x) = x^3$.

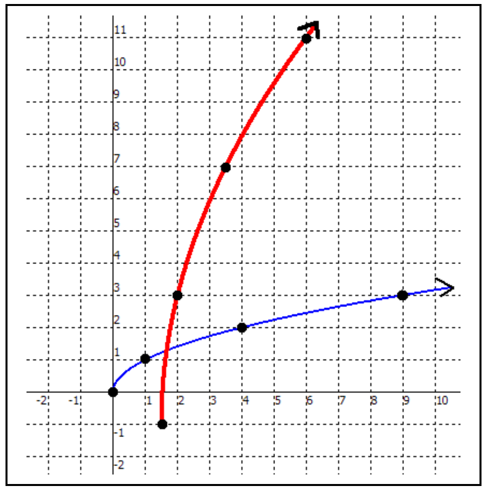
1st Translate (Shift or Slide) the points of the parent function 5 units to the right.

2nd Reflect the points of the graph over the y-axis, then Compress (Squeeze) them horizontally to positions $\frac{3}{4}$ as far from the y-axis.

3rd Compress (Squeeze) the points of the graph vertically toward the x-axis to positions $\frac{1}{2}$ as far from the x-axis.

4th Translate (Shift or Slide) the points of the graph 6 units up.

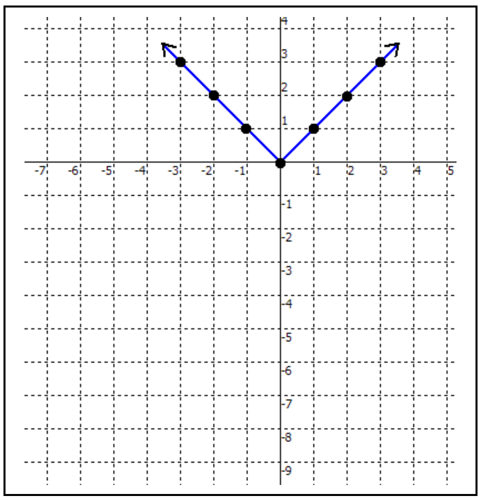


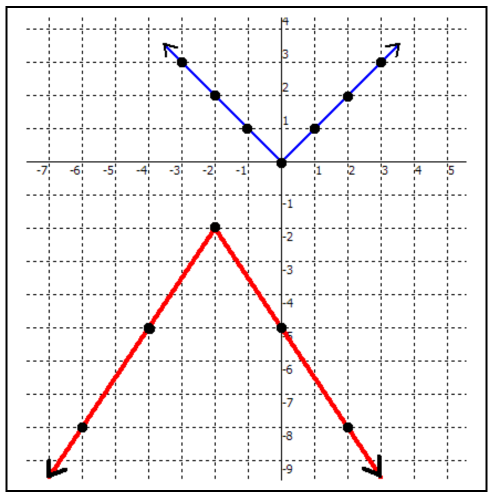


$$y = 4\sqrt{2x - 3} - 1$$

$$y = \sqrt{x}$$

$\div 2 \swarrow$	$+3 \swarrow$	x	y	$\searrow \cdot 4$	$\searrow -1$
$1\frac{1}{2}$	3	0	0	0	−1
2	4	1	1	4	3
$3\frac{1}{2}$	7	4	2	8	7
6	12	9	3	12	11
$9\frac{1}{2}$	19	16	4	16	15

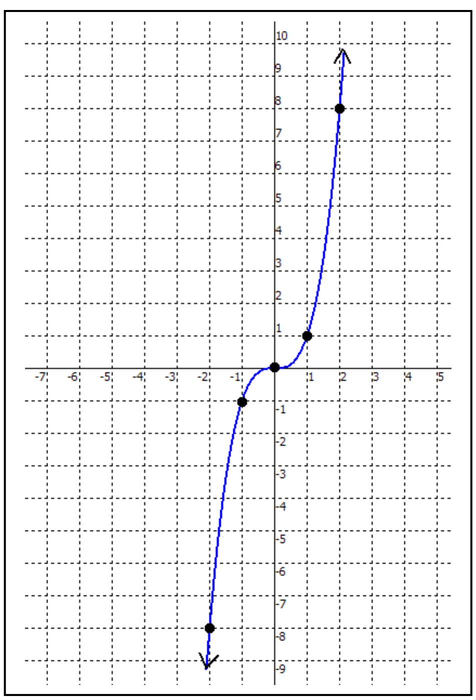


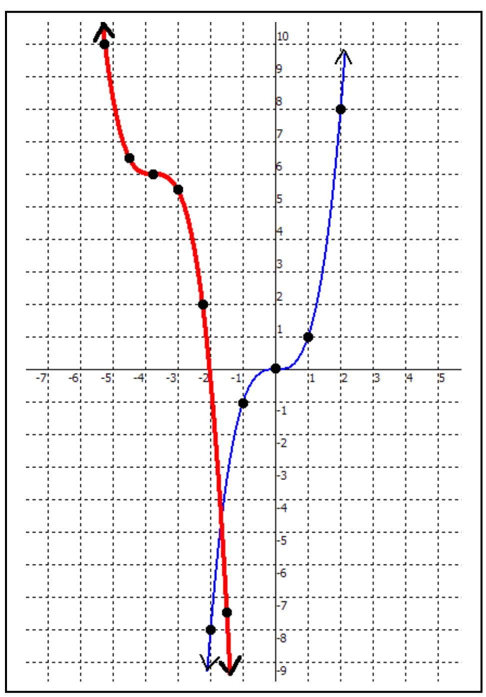


$$y = -3\left|\frac{1}{2}x + 1\right| - 2$$

$$y = |x|$$

$\cdot 2 \swarrow$	$-1 \swarrow$	x	y	$\searrow \cdot -3$	$\searrow -2$
−2	−1	0	0	0	−2
−4	−2	−1	1	−3	−5
0	0	1	1	−3	−5
−6	−3	−2	2	−6	−8
2	1	2	2	−6	−8
−8	−4	−3	3	−9	−11
4	2	3	3	−9	−11

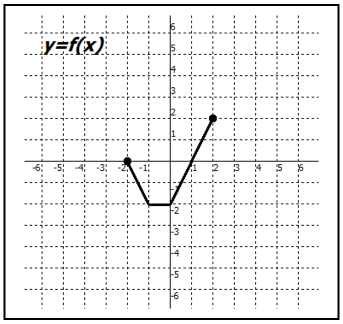


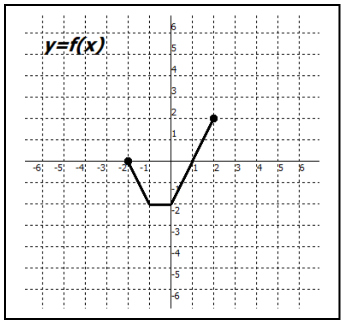


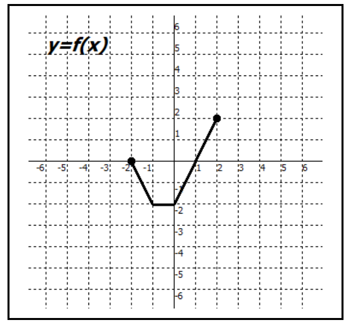
$$y = \frac{1}{2}\left(-\frac{4}{3}x - 5\right)^3 + 6$$

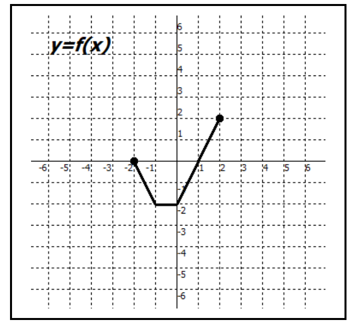
$$y = (x)^3$$

$\cdot -\frac{3}{4} \swarrow$	$+5 \swarrow$	x	y	$\searrow \cdot \frac{1}{2}$	$\searrow +6$
$-3\frac{3}{4}$	5	0	0	0	6
−3	4	−1	−1	$-\frac{1}{2}$	$5\frac{1}{2}$
$4\frac{1}{2}$	6	1	1	$\frac{1}{2}$	$6\frac{1}{2}$
$-2\frac{1}{4}$	3	−2	−8	−4	2
$-5\frac{1}{4}$	7	2	8	4	10
$-1\frac{1}{2}$	2	−3	−27	$-\frac{27}{2}$	$-7\frac{1}{2}$
−6	8	3	27	$\frac{27}{2}$	$19\frac{1}{2}$









A. $y = f(x) + 3$

B. $y = f(x+4)$

C. $y = f(x) - 4$

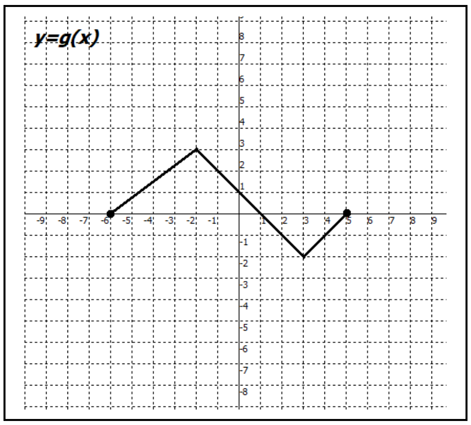
D. $y = f(x-2)$

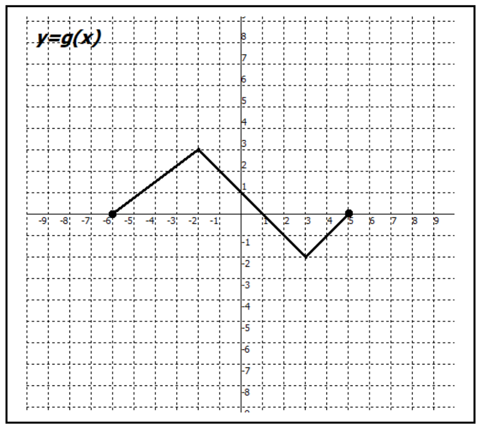
E. $y = f(x+3) + 4$

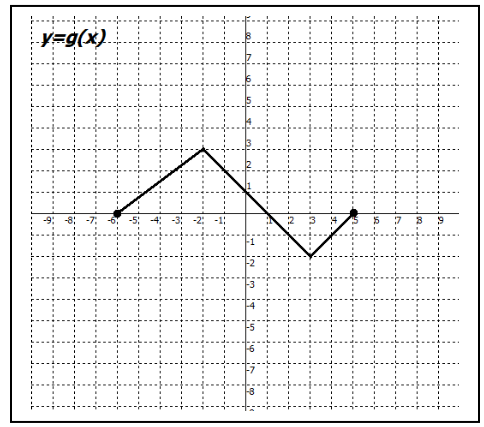
F. $y = f(x-1) - 3$

G. $y = f(x-4) + 1$

H. $y = f(x+2) - 1$







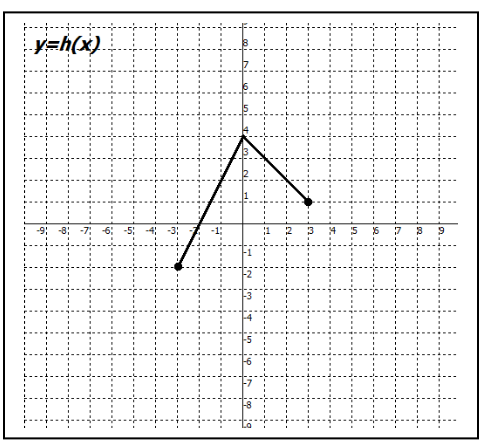
J. $y = 2g(x)$

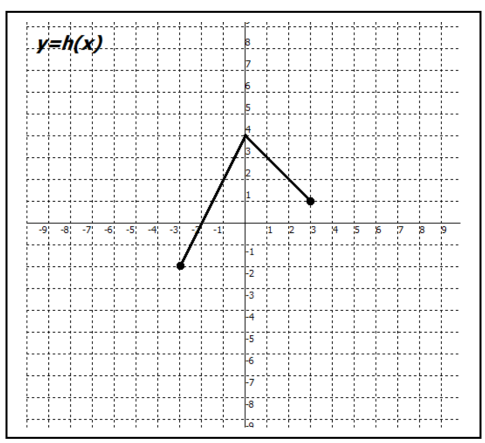
K. $y = \frac{1}{3}g(x)$

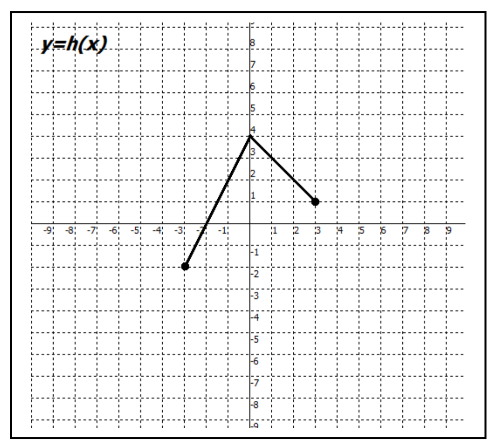
L. $y = 3g(x)$

M. $y = \frac{1}{2}g(x)$

N. $y = -g(x)$







P. $y = h(3x)$

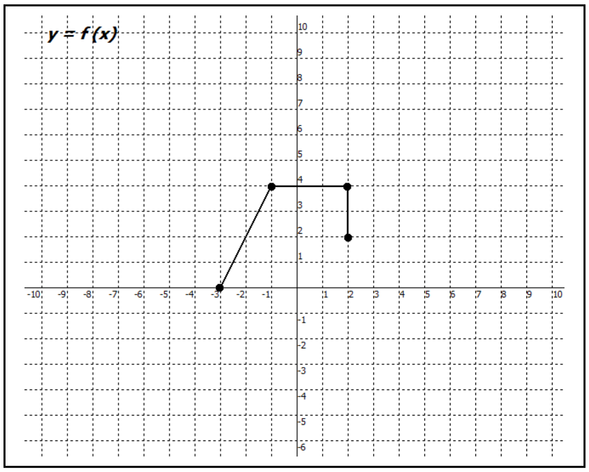
Q. $y = h\left(\frac{1}{2}x\right)$

R. $y = h(2x)$

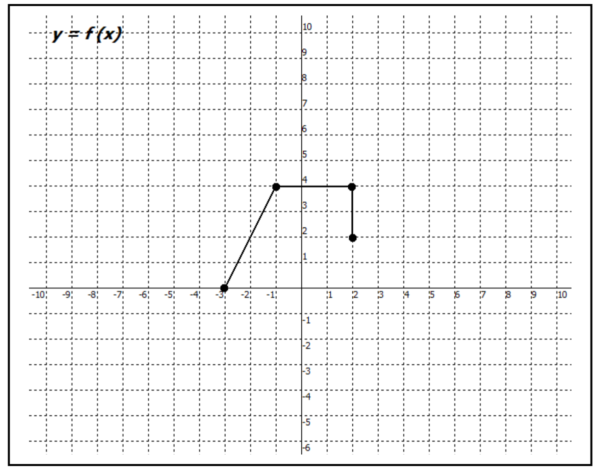
S. $y = h\left(\frac{1}{3}x\right)$

T. $y = h(-x)$

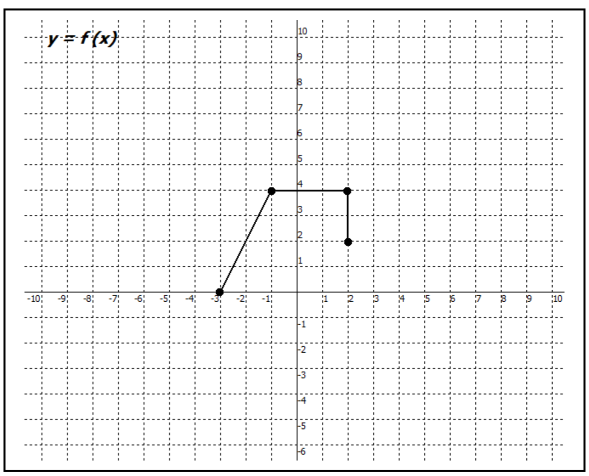
Graph: $y = f\left(\frac{1}{3}x\right) + 4$



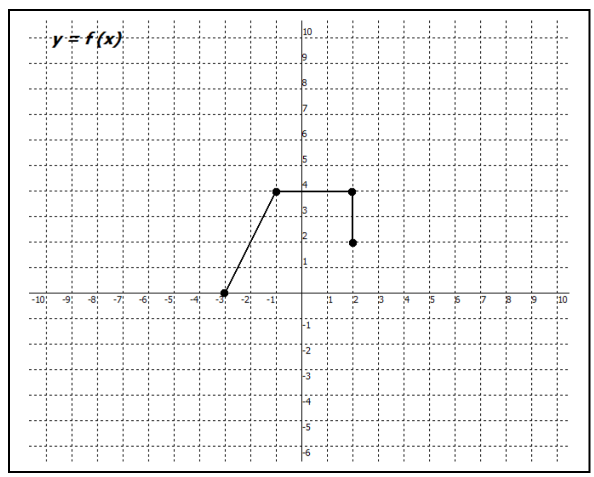
Graph: $y = -2f(x-5)$



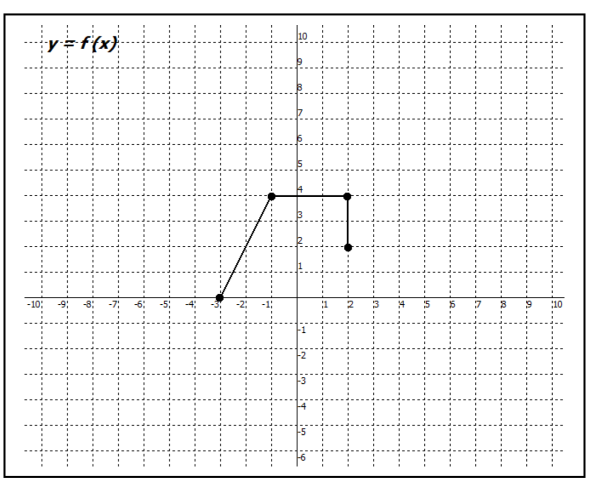
Graph: $y = f(-x) - 5$



Graph: $y = \frac{1}{2}f(x-6) + 3$



Graph: $y = -\frac{2}{3}f(x+1) - 3$



Graph: $y = 2f[-x-7] + 1$