MATH 241 Thomas 13th Edition Name______________

Chapter 15, Section 7

Cylindrical coordinates represent a point P in space by ordered triples (r,θ,z) in which $r \geq 0$, where r and θ are polar coordinates for the vertical projection of P on the xy-plane and z is the rectangular vertical coordinate.

Equations relating rectangular and cylindrical coordinates

$$\lim_{n\to\infty} S_n = \iiint_D f\, dV = \iiint_D f\, dz\, r\, dr\, d\theta$$

How to integrate in cylindrical form: sketch the graph, find z limits of integration then r limits and finally theta limits then compute the value.

Spherical coordinates represent a point P in space by ordered triples (ρ,ϕ,θ) in which ρ is the distance from P to the origin $\rho \geq 0$, ϕ is the angle $\overrightarrow{OP}$ makes with the positive z-axis and θ is the angle from cylindrical coordinates.

Equations relating spherical coordinates to Cartesian and cylindrical coordinates

$$\lim_{n\to\infty} S_n = \iiint_D f(\rho,\phi,\theta)\, dV = \iiint_D f(\rho,\phi,\theta)\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

Conversion Formulas

$$\begin{array}{ccc} x = r\cos\theta & x = \rho\sin\phi\cos\theta & r = \rho\sin\phi \\ y = r\sin\theta & y = \rho\sin\phi\sin\theta & z = \rho\cos\phi \\ z = z & z = \rho\cos\phi & \theta = \theta \end{array}$$

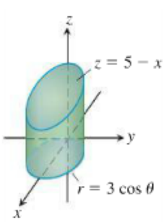
Formulas for *dV* in triple integrals

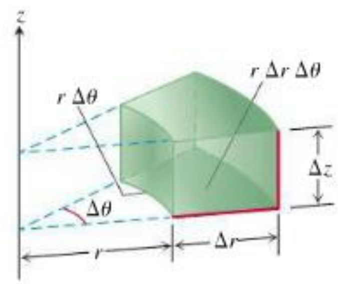
$$\begin{aligned} dV = \quad & dx\,dy\,dz \\ & dz\, r\, dr\, d\theta \\ & \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta \end{aligned}$$

Evaluate

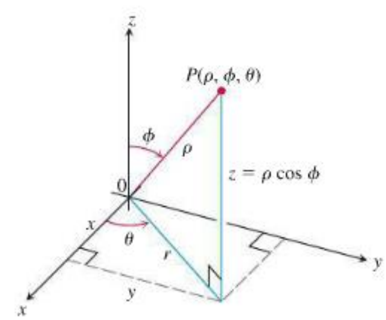
$$\int_0^{2\pi}\int_0^{1}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(r^2\sin^2\theta+z^2\right)dz\,r\,dr\,d\theta$$

Set up the integral for the right circular cylinder whose base is the circle $r=3\cos\theta$ and whose top lies in the plane $z=5-x$





Evaluate $\int_0^{\frac{3\pi}{2}}\int_0^{\pi}\int_0^{1}5\rho^3\sin^3\phi\,d\rho\,d\phi\,d\theta$



Find the spherical integration and evaluate for the volume of $\rho=1,\ z\geq 0,$ and above by the cardioid of revolution $\rho=1+\cos\phi$