MATH 241 Thomas 13th Edition Name_______________

Chapter 15, Section 6

Moments and Center of Masses

The first moment of a solid region D about a coordinate plane is defined as the triple integral over D of the distance from a point (x, y, z) in D to the plane multiplied by the density of the solid at that point.

$$M_{xy} = \iiint_D z\delta(x,y,z)dV \qquad M_{xz} = \iiint_D y\delta(x,y,z)dV \qquad M_{yz} = \iiint_D x\delta(x,y,z)dV$$

The center of mass is found from the first moments, $(\bar{x},\bar{y},\bar{z}) = \left(\frac{M_{yz}}{M}, \frac{M_{xz}}{M}, \frac{M_{xy}}{M}\right)$

When the density of a solid object or plate is a constant, the center of mass is called the centroid of the object.

An object's first moments tell us about balance and about the torque the object experiences about different axes in a gravitational field. The second moment or moment of inertia is used for looking at how much energy is generated.

THREE-DIMENSIONAL SOLID

Mass: $M = \iiint_D \delta\, dV$ $\quad \delta = \delta(x, y, z)$ is the density at (x, y, z).

First moments about the coordinate planes:

$$M_{yz} = \iiint_D x\,\delta\, dV, \quad M_{xz} = \iiint_D y\,\delta\, dV, \quad M_{xy} = \iiint_D z\,\delta\, dV$$

Center of mass:

$$\bar{x} = \frac{M_{yz}}{M}, \quad \bar{y} = \frac{M_{xz}}{M}, \quad \bar{z} = \frac{M_{xy}}{M}$$

TWO-DIMENSIONAL PLATE

Mass: $M = \iint_R \delta\, dA$ $\quad \delta = \delta(x, y)$ is the density at (x, y).

First moments: $M_y = \iint_R x\,\delta\, dA, \quad M_x = \iint_R y\,\delta\, dA$

Center of mass: $\bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}$

THREE-DIMENSIONAL SOLID

About the x-axis: $I_x = \iiint (y^2 + z^2)\,\delta\, dV$ $\quad \delta = \delta(x, y, z)$

About the y-axis: $I_y = \iiint (x^2 + z^2)\,\delta\, dV$

About the z-axis: $I_z = \iiint (x^2 + y^2)\,\delta\, dV$

About a line L: $I_L = \iiint r^2(x, y, z)\,\delta\, dV$ $\quad r(x, y, z)$ = distance from the point (x, y, z) to line L

TWO-DIMENSIONAL PLATE

About the x-axis: $I_x = \iint y^2\,\delta\, dA$ $\quad \delta = \delta(x, y)$

About the y-axis: $I_y = \iint x^2\,\delta\, dA$

About a line L: $I_L = \iint r^2(x, y)\,\delta\, dA$ $\quad r(x, y)$ = distance from (x, y) to L

About the origin (polar moment): $I_0 = \iint (x^2 + y^2)\,\delta\, dA = I_x + I_y$

Find the moments of inertia about the coordinate axes of a think rectangular plate of constant density bounded by the lines x = 3 and y = 3 in the first quadrant.

Find the centroid of the triangular region cut from the first quadrant by the line $x+y=3$

Find the first moment about the y-axis of a thin plate of density 1 covering the infinite region under the curve $y=e^{\frac{-x^2}{2}}$ in the first quadrant.

Find the mass of a thin plate occupying the smaller region cut from the ellipse $x^2+4y^2=12$ by the parabola

$x=4y^2$ if $\delta(x,y)=5x$.