MATH 241 Thomas 13th Edition Name______________

Chapter 15, Section 5

Triple integrals in Rectangular Coordinates are used to calculate volumes of three-dimensional shapes, average value of a function over a three-dimensional region. Other applications are vector fields and fluid flow in three dimensions.

The volume of a closed, bounded region D in space is

Finding the limits of integration in the order $dzdydx$

1. Graph
2. Find the z-limits by drawing a line through a point (x, y) parallel to the z-axis
3. Find the y-limits by drawing a line through a point (x, y) parallel to the y-axis
4. Find the x-limits

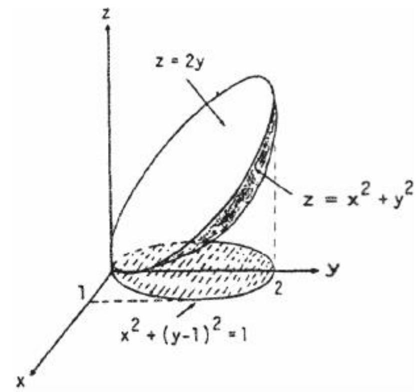
The integral is $\int_{x=a}^{x=b} \int_{y=g_1(x)}^{y=g_2(x)} \int_{z=f_1(x,y)}^{z=f_2(x,y)} F(x,y,z)dzdydx$

Examples:

Volume of rectangular solid Write six different iterated triple integrals for the volume of the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 2$, and $z = 3$. Evaluate one of the integrals.

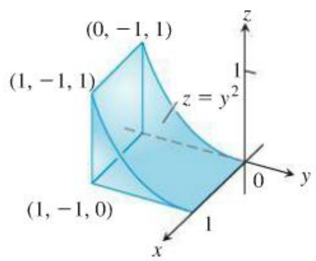
Volume of solid Write six different iterated triple integrals for the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$. Evaluate one of the integrals.

Volume inside paraboloid beneath a plane Let D be the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$. Write triple iterated integrals in the order $dz\,dx\,dy$ and $dz\,dy\,dx$ that give the volume of D. Do not evaluate either integral.



Here is the region of integration of the integral

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dz\,dy\,dx.$$



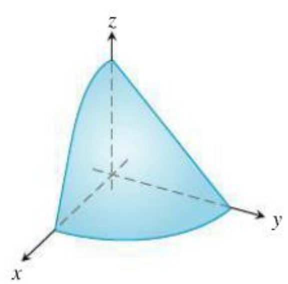
Rewrite the integral as an equivalent iterated integral in the order

a. $dy\,dz\,dx$ **b.** $dy\,dx\,dz$

c. $dx\,dy\,dz$ **d.** $dx\,dz\,dy$

e. $dz\,dx\,dy$.

. The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$



Find Average value:

. $F(x, y, z) = xyz$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$, and $z = 2$