

Chapter 15, Section 4

Integrals in Polar Coordinates use “polar rectangles”

Area of a circle

Area of a sector

Area of a small slice of a sector

Area of a region in Polar Coordinates

Examples:

Find the interval for theta and r when the region is enclosed by the semicircle

$$x^2 + y^2 = 2y, y \geq 0$$

Change the Cartesian integral to Polar and then evaluate

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$$

$$\int_0^2 \int_0^x y dy dx$$

$$\int_{\sqrt{2}}^2 \int_{\sqrt{4-y^2}}^y dx dy$$

Sketch the region and set up an equivalent in Cartesian

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_1^{\csc \theta} r^2 \cos \theta dr d\theta$$

Find the area of the region inside $r = 1 + \cos \theta$ and outside $r = 1$