MATH 241 Thomas 13th Edition Name____________________
Chapter 15, Section 8

DEFINITION The **Jacobian determinant** or **Jacobian** of the coordinate transformation $x = g(u, v)$, $y = h(u, v)$ is

$$J(u, v) = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial y}{\partial u}\frac{\partial x}{\partial v}.$$

THEOREM —Substitution for Double Integrals

Suppose that $f(x, y)$ is continuous over the region R. Let G be the preimage of R under the transformation $x = g(u, v)$, $y = h(u, v)$, which is assumed to be one-to-one on the interior of G. If the functions g and h have continuous first partial derivatives within the interior of G, then

$$\iint_R f(x, y)\,dx\,dy = \iint_G f(g(u, v), h(u, v))\left|\frac{\partial(x, y)}{\partial(u, v)}\right| du\,dv.$$

Examples:

Solve the system

$$u = 2x - 3y, \qquad v = -x + y$$

For x and y in terms of u and v. Then find the value of the Jacobian $\partial(x, y)/\partial(u, v)$.
Find the image under the transformation $u = 2x - 3y, v = -x + y$ of the parallelogram R in the xy-plane with boundaries $x = -3, x = 0, y = x$, and $y = x + 1$.
Sketch the transformed region in the uv-plane.

Use the Transformation and parallelogram R to evaluate

$$\iint_R 2(x - y)\, dxdy$$