

## Chapter 14, Section 8

## Lagrange multiplier

The extreme values of a function  $f(x, y, z)$  whose variables are subject to a constraint  $g(x, y, z) = 0$  are found on the surface \_\_\_\_\_ among the points where \_\_\_\_\_ for some scalar \_\_\_\_\_ .

## Orthogonal Gradient Theorem

Suppose that  $f(x, y, z)$  is differentiable in a region whose interior contains a smooth curve:

If  $P_0$  is a point on the curve where  $f$  has a local max or local min relative to its values on the curve then

Suppose  $f(x, y, z)$  and  $g(x, y, z)$  are differentiable and  $\nabla g \neq 0$  when  $g(x, y, z) = 0$  then to find the local max and local min values of  $f$  subject to the constraints of  $g(x, y, z) = 0$ , find the values of  $x, y, z$  and  $\lambda$  that simultaneously satisfy

Examples:

$$f(x, y) = xy, \quad g(x, y) = x^2 + y^2 - 10 = 0$$

$$x^2 y = 2, \text{ nearest the origin}$$

Cylinder in a sphere

$x^2 + y^2 + z^2 = 4$  farthest from the point  $(1, -1, 1)$

$x^2 + y^2 + z^2 = 25$  where  $f(x, y, z) = x + 2y + 3z$  has its max and min values