MATH 241 Thomas 13th Edition Name______________

Chapter 14, Section 7

Let $f(x,y)$ be defined on a region R containing the point (a,b), then (a,b) is a local max value of $f(x,y)$ if

and a local min if

If $f(x,y)$ has a local max or min value at an interior point (a,b) of its domain and if the first partial derivative exists there, then

An interior point of the domain of a function $f(x,y)$ where both f_x and f_y are zero or where one or both do not exist is a critical point.

A differentiable funtion $f(x,y)$ has a saddle point at a critical point (a,b) if in every open disk centered at (a,b) there are domain points (x,y) where and domain points (x,y) where . The point $(a,b,f(a,b))$ on the surface of $z=f(x,y)$ is a saddle point.

Summary of Max-Min Tests

The extreme values of $f(x, y)$ can occur only at

i) **boundary points** of the domain of f

ii) **critical points** (interior points where $f_x = f_y = 0$ or points where f_x or f_y fails to exist).

If the first- and second-order partial derivatives of f are continuous throughout a disk centered at a point (a, b) and $f_x(a, b) = f_y(a, b) = 0$, the nature of $f(a, b)$ can be tested with the **Second Derivative Test**:

i) $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}{}^2 > 0$ at $(a, b) \Rightarrow$ **local maximum**

ii) $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}{}^2 > 0$ at $(a, b) \Rightarrow$ **local minimum**

iii) $f_{xx}f_{yy} - f_{xy}{}^2 < 0$ at $(a, b) \Rightarrow$ **saddle point**

iv) $f_{xx}f_{yy} - f_{xy}{}^2 = 0$ at $(a, b) \Rightarrow$ **test is inconclusive**

Relationship between discriminant and determinant

Examples: (write small!!!)

$f(x,y) = 5xy - 7x^2 + 3x - 6y + 2$

$$f(x,y)=x^2-2xy+2y^2-2x+2y+1$$

$$f(x,y)=1-\sqrt[3]{x^2+y^2}$$

$$f(x,y)=x^3+y^3+3x^2-3y^2-8$$

$$f(x,y)=48xy-32x^3-24y^2,\ \ 0\le x\le 1,\ \ 0\le y\le 1$$

$$f(x,y)=x^2-xy+y^2+1,\ \ x=0,\ \ y=4,\ \ y=x$$