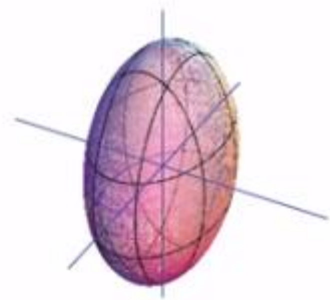
MATH 241, Chapter 12 section 6 Thomas 13th Edition Name___________________

A quadric surface is the graph in space of a second degree equation in x, y, and x. Quadric surfaces are given by the general equation:

The basic quadric surfaces are:

Ellipsoids

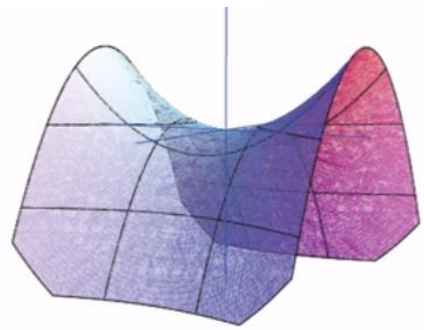
The ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ cuts the coordinate axes at the points $(\pm a, 0, 0)$, $(0, \pm b, 0)$, $(0, 0, \pm c)$. Let's look at a picture:



Hyperbolic Paraboloids

The **hyperbolic paraboloid** $\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$, $c > 0$,

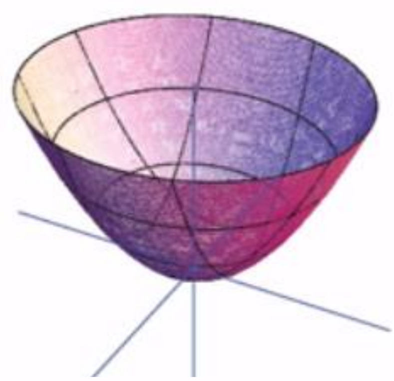
has symmetry with respect to the planes $x = 0$ and $y = 0$. The cross-sections in these planes are parabolas, as shown below:

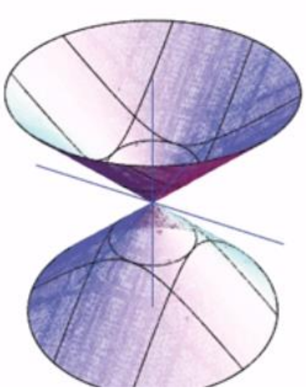


Elliptical Paraboloids

The **elliptical paraboloid** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

has symmetry with respect to the planes $x = 0$ and $y = 0$. Again, the cross-sections in these planes are parabolas. In the plane $z = c$, we have an ellipse. This is illustrated in the following graph:





Elliptical Cones

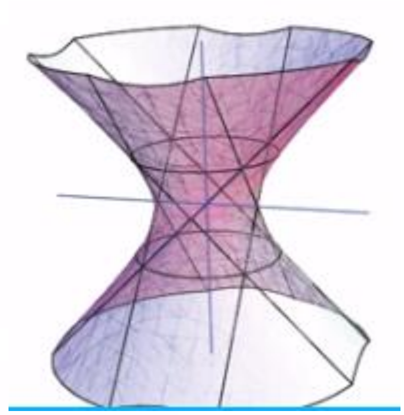
The **elliptical cone** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

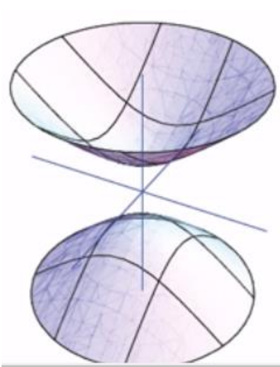
has symmetry with respect to the z-axis. In the xz- and yz-planes, the cross sections are lines. In the plane $z = c$, the cross section is an ellipse. Look at the graph below:

Hyperboloids of One Sheet

The **hyperboloid of one sheet** $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

has symmetry with respect to the z-axis. The cross-sections in the xz- and yz-planes are hyperbolas. In the plane $z = c$, we have an ellipse. Here's a picture:





Hyperboloids of Two Sheets

The **hyperboloid of two sheets** $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

has symmetry with respect to the z-axis. The cross-sections in the xz- and yz-planes are again hyperbolas. In the plane $z = c$, the cross section is an ellipse. Let's look at a graph:

In the six basic types of quadric surfaces given above, each one is symmetric with respect to the z-axis. However, with appropriate changes to each equation, any coordinate axis can serve as an axis of symmetry.

$4y^2 + z^2 = 9$ $\qquad x^2 + y^2 + 9z^2 = 36$ $\qquad z = 4y^2 - 1$