

One-To-One Functions

A function  $f$  is *one-to-one* if different inputs have different outputs. That is, if for  $a$  and  $b$  in the domain of  $f$  with  $a \neq b$ , we have  $f(a) \neq f(b)$ , then the function  $f$  is one-to-one. If a function is one-to-one, then its inverse correspondence is also a function.

The Horizontal Line Test

If it is impossible to draw a horizontal line that intersects a function's graph more than once, then the function is one-to-one. For every one-to-one function, an inverse function exists.

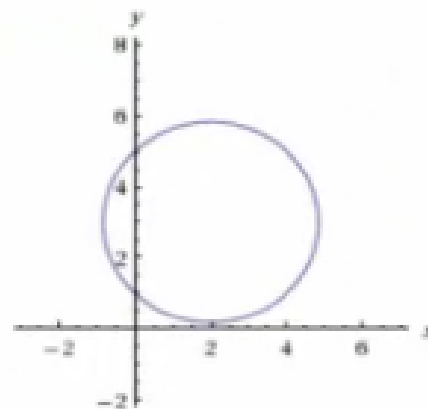
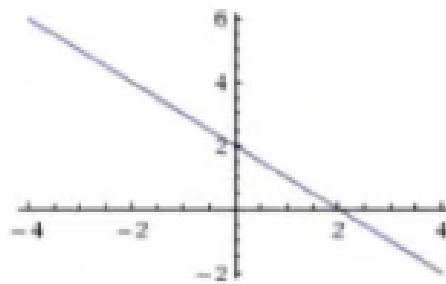
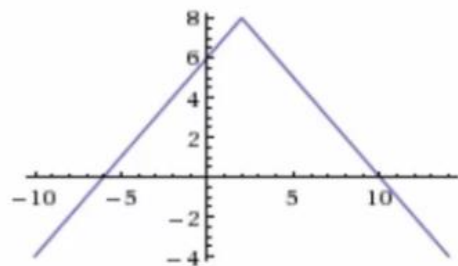
To Find a Formula for  $f^{-1}$ 

First make sure that  $f$  is one-to-one. Then replace  $f(x)$  with  $y$ . Next interchange  $x$  and  $y$ . (This gives the inverse function.) Solve for  $y$ . Finally, replace  $y$  with  $f^{-1}(x)$ . (This is inverse function notation.)

Composition of Inverse Functions Property

$$f^{-1}[f(x)] = x \text{ and } f[f^{-1}(x)] = x$$

- Do these represent One-to-One Functions?



- Determine whether the function is one-to-one; if it is one-to-one, find a formula for the inverse and graph both on the same set of axis..

1.  $g(x) = 2x - 3$

2.  $f(x) = x^2 - 4$

3.  $H(x) = 2$

4.  $g(x) = 5x + 15$

5.  $\{(-5, -2), (-3, 2), (-1, 4), (1, 5)\}$

6.  $\{(-5, -2), (-5, 2), (-1, 4), (1, 5)\}$

7.  $\{(-5, -2), (-5, 2), (-1, 5), (1, 5)\}$

Determine if the functions are inverses of each other.

8.  $h(x) = 5x - 5; f(x) = \frac{1}{5}x + \frac{1}{5}$

9.  $h(x) = 3x; f(x) = \frac{1}{3x}$

10. Given  $f(x) = 2x - 4$ , find  $f^{-1}(5)$