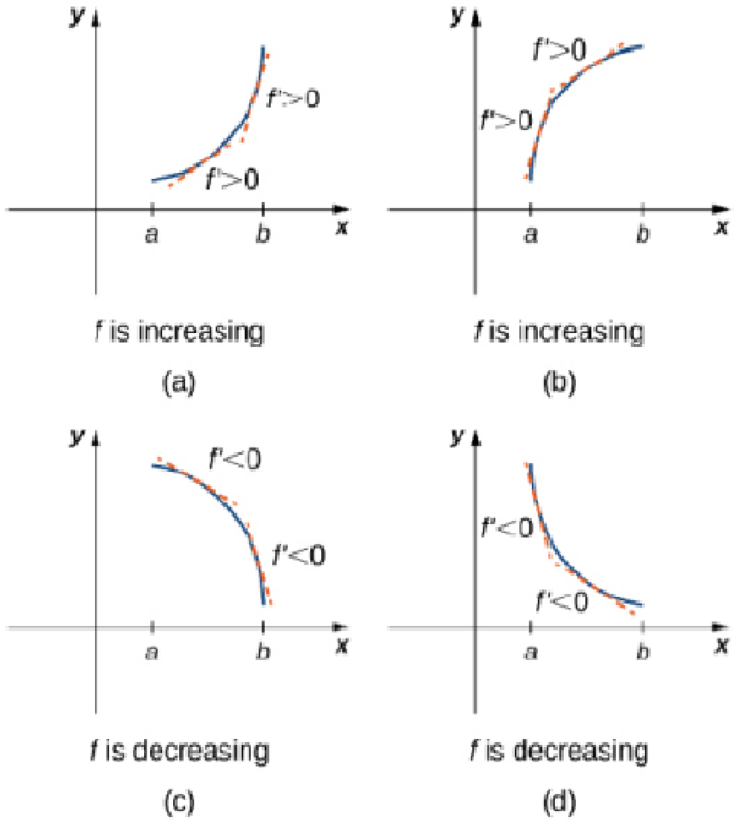
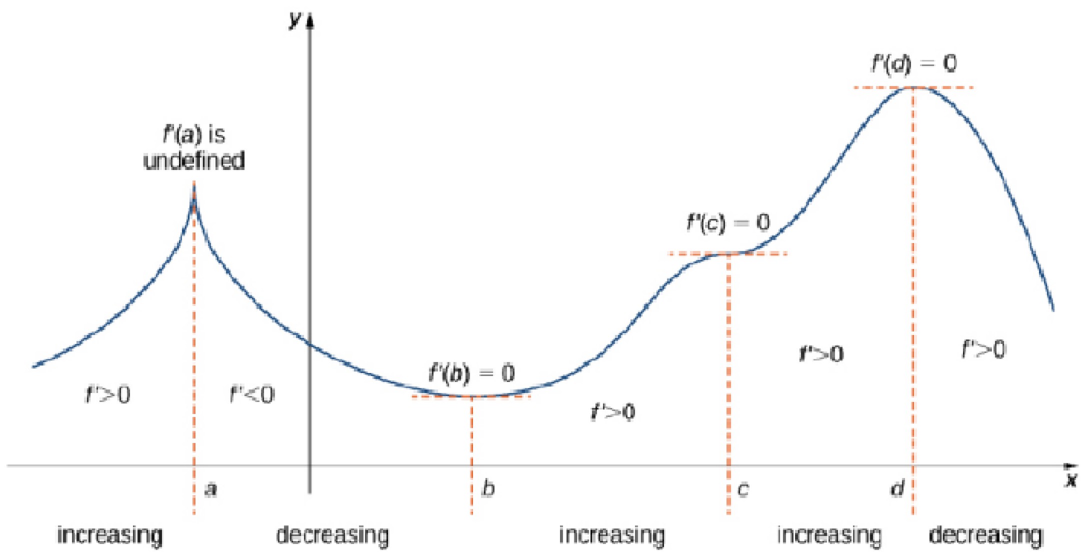
Math 141 - Calculus
Section 4.5 Video Worksheet

Name___________________




## The First Derivative Test

- If a _____________ function $f$ has a local extremum, it must occur at a critical point _____.
- The function has a local _____________ at the critical point $c$ if and only if the derivative $f'$ _____________ sign as $x$ increases through $c$.
- Therefore, to test whether a function has a local extremum at a critical point $c$, we must determine the ________ of ______ to the left and right of $c$.

### Theorem 4.9: First Derivative Test

Suppose that $f$ is a continuous function over an interval $I$ containing a critical point $c$. If $f$ is differentiable over $I$, except possibly at point $c$, then $f(c)$ satisfies one of the following descriptions:

i. If $f'$ changes sign from positive when $x < c$ to negative when $x > c$, then $f(c)$ is a local maximum of $f$.

ii. If $f'$ changes sign from negative when $x < c$ to positive when $x > c$, then $f(c)$ is a local minimum of $f$.

iii. If $f'$ has the same sign for $x < c$ and $x > c$, then $f(c)$ is neither a local maximum nor a local minimum of $f$.

### Definition

Let $f$ be a function that is differentiable over an open interval $I$. If $f'$ is increasing over $I$, we say $f$ is **concave up** over $I$. If $f'$ is decreasing over $I$, we say $f$ is **concave down** over $I$.

### Theorem 4.10: Test for Concavity

Let $f$ be a function that is twice differentiable over an interval $I$.

i. If $f''(x) > 0$ for all $x \in I$, then $f$ is concave up over $I$.

ii. If $f''(x) < 0$ for all $x \in I$, then $f$ is concave down over $I$.


### Definition

Let $f$ be a function that is differentiable over an open interval $I$. If $f'$ is increasing over $I$, we say $f$ is **concave up** over $I$. If $f'$ is decreasing over $I$, we say $f$ is **concave down** over $I$.


### Theorem 4.11: Second Derivative Test

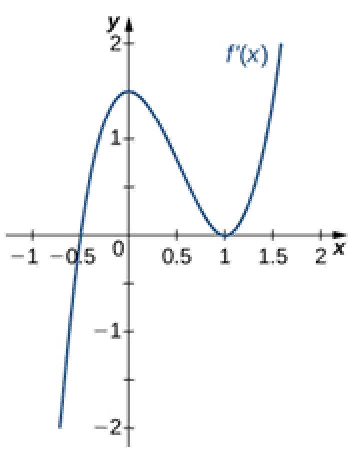
Suppose $f'(c) = 0$, $f''$ is continuous over an interval containing $c$.

i. If $f''(c) > 0$, then $f$ has a local minimum at $c$.

ii. If $f''(c) < 0$, then $f$ has a local maximum at $c$.

iii. If $f''(c) = 0$, then the test is inconclusive.

For the following find where the graph is increasing, decreasing, maximum, minimums, concave up and concave down along with points of inflection.




$$f(x) = x^4 - 6x^3$$