

One-sided Limits

To have a limit L as x approaches c ,

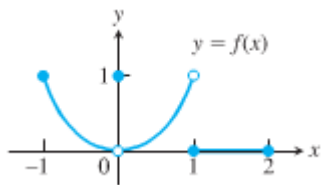
If f fails to have a two-sided limit at c ,

A function $f(x)$ has a limit as x approaches c iff

Theorem:

$$\lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0$$

Proof:



True or False?

$$\lim_{x \rightarrow -1^+} f(x) = 1$$

$$\lim_{x \rightarrow -0^-} f(x) = 1$$

$$\lim_{x \rightarrow 0} f(x) \text{ exists}$$

$$\lim_{x \rightarrow 0} f(x) = 1$$

$$\lim_{x \rightarrow 1} f(x) = 1$$

$$\lim_{x \rightarrow -1^-} f(x) \text{ does not exist}$$

Try It:

$$\lim_{x \rightarrow 2^-} f(x) = 2$$

$$\lim_{t \rightarrow 0} \frac{\sin(kt)}{t} =$$

$$\lim_{t \rightarrow 0} \frac{t}{\sin t} =$$

$$\lim_{h \rightarrow 0} \frac{h}{\sin(3h)} =$$

$$\lim_{x \rightarrow 0} 6x^2 (\cot x) (\csc(2x)) =$$

$$\lim_{t \rightarrow 0} \frac{2t}{\tan t} =$$

$$\lim_{y \rightarrow 0} \frac{\sin(\sin h)}{\sin h} =$$

$$\lim_{y \rightarrow 0} \frac{\sin(3y) \cot(5y)}{y \cot(4y)} =$$

$$\lim_{x \rightarrow 0} \frac{x + x \cos x}{\sin x \cos x} =$$

$$\text{Try It: } \lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} =$$