

Rotation of Axes

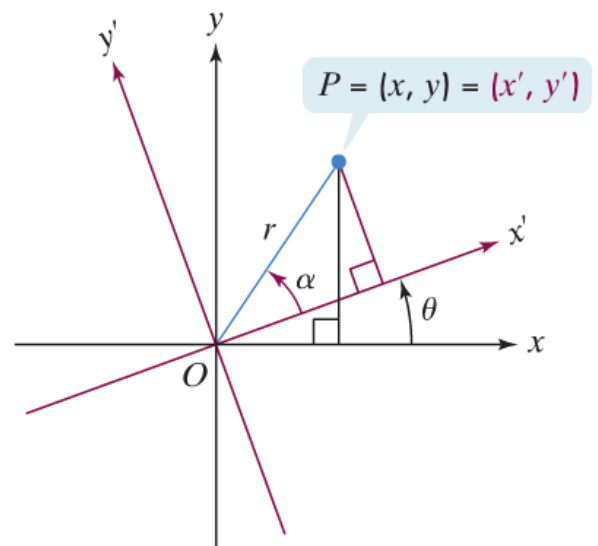
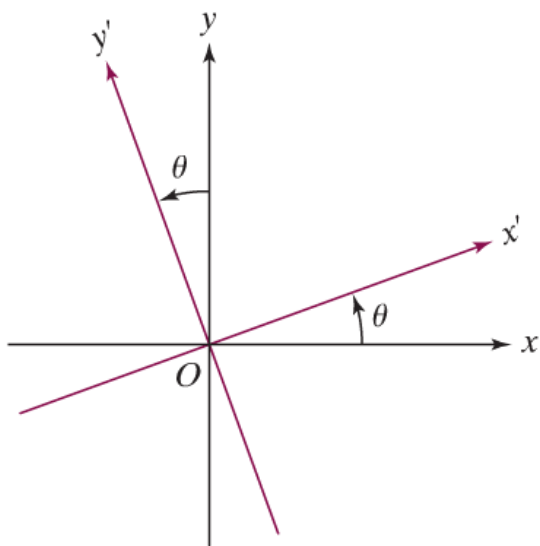
Identifying a Conic Section without Completing the Square

$$y^2 - 4x + 2y + 21 = 0$$

$$4x^2 - 9y^2 - 8x - 36y - 68 = 0$$

$$4x^2 + 4y^2 + 12x + 4y + 1 = 0$$

$$9x^2 + 4y^2 - 36x + 8y + 31 = 0$$



$$xy = -4 \quad \theta = 45^\circ$$

$$x = x' \cos 45^\circ - y' \sin 45^\circ \quad y = x' \sin 45^\circ + y' \cos 45^\circ$$

Amount of Rotation Formula

The general second-degree equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0, B \neq 0$$

can be rewritten as an equation in x' and y' without an $x'y'$ -term by rotating the axes through angle θ , where

$$\cot 2\theta = \frac{A - C}{B}.$$

Writing the Equation of a Rotated Conic in Standard Form

$$x^2 - \sqrt{3}xy + 2y^2 = 1$$

If $\cot(2\theta)$ is not a familiar angle then

Identifying a Conic Section without a Rotation of Axes

$$x^2 - 3xy + y^2 - x = 0$$

$$3x^2 - 18xy + 27y^2 - 5x + 7y = -4$$

$$2x^2 - \sqrt{15}xy + 2y^2 + x + y = 0$$